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THE SPIRAL NEBULA *H I, 55 PEGASI.*

By JAMES E. KEELER.

THE small nebula *H I, 55 Pegasi* (*G. C.* 4892) is not in itself very remarkable, as compared with many other objects of the same class, but considerable interest attaches to it on account of the diverse forms which have been ascribed to it by different observers.

Herschel describes this nebula as pretty bright; irregularly round; resolvable; elongated between two stars; containing two or three stars. In another observation it is described as much elongated, its length being $2'.1$ and its breadth $30''$. The drawing[1] shows a narrow, spindle-shaped nebula, which accords with this description.

D'Arrest's drawing[2] much resembles that of Herschel, but the width of the nebula is greater, and the condensation toward the center is shown. The conventionalized, lozenge-shaped outline gives the drawing a somewhat unnatural appearance.

[1] *Phil. Trans.*, 1833, Pl. XIV, Fig. 39.

[2] *Instrumentum magnum aequatoreum*, Pl. II, Fig. 6.

Lord Rosse, who observed the nebula many times, was uncertain whether to class it as an annular or a spiral nebula. His drawing[1] shows a spindle-shaped nebulous mass, somewhat like Herschel's, with spiral convolutions surrounding a star of about the fifteenth magnitude on the preceding side.

In Volume I of *Himmel und Erde* there is a drawing[2] by Tempel, which was communicated to that journal with some interesting remarks on the subject of unconscious personal tendencies in drawing faint objects. All the above-mentioned drawings are brought together, for comparison, in Plate III, and they are reproduced here with the acknowledgments of the Editors of the ASTROPHYSICAL JOURNAL.

In Tempel's drawing the spiral convolutions of Lord Rosse are represented by a diffuse patch of nebulosity, in which no structure is shown. The writer of the article in *Himmel und Erde* observes that Tempel's failure to see a spiral form could not have been due to insufficient optical power, since he has shown a faint new nebula which escaped the telescope of Lord Rosse; but this remark loses all its force when it is shown (as it is shown farther below), that Tempel's supposed nebula certainly does not exist. There is no doubt that Lord Rosse saw the nebula to much greater advantage than Tempel.

A photograph of this nebula has been made by Dr. Roberts, and is reproduced on plate 50 of his *Selection of Photographs of Stars, Star-Clusters and Nebulae*. The description there given is as follows:

"The photograph shows the nebula to be elliptical, with a dense, broad line of nebulosity, curved at both ends, forming the major axis, which has a star of about fifteenth magnitude in its center, and there is also a slightly fainter star in the center of the *preceding* semi-ellipse. No structure is visible in it, such as that shown on the drawing by Lord Rosse, but the semi-ellipse on the *following* side is shown on the photograph, though it is not on the drawing."

[1] *Phil. Trans.*, 1850, Pl. XXXVI, Fig. 4.

[2] Opp., p. 133.

So far as the general character of the nebula is concerned, Dr. Roberts seems to have misinterpreted the details shown by his photograph, owing doubtless to the small scale of the negative. The exposure was amply sufficient to bring out the fainter parts of the nebula.

Two photographs of *H I, 55 Pegasi* have been taken with the Crossley reflector, which has a focal length more than twice as great as that of Dr. Roberts' telescope, and is correspondingly better adapted to the photography of these small nebulae. A plate was exposed on August 9, 1899, for two hours, and another on August 28 for three hours and thirty minutes, yielding an excellent negative. From this a positive was made on glass, with an enlargement of 7.5 diameters. It is reproduced in the accompanying plate. The slight ellipticity of the star-disks is in this case due, not to imperfect guiding, but to aberration, the nebula having been photographed at some little distance from the center of the plate.

A glance at the photograph shows that the nebula is a two-branched, left-handed spiral, with a nucleus or condensation near the point of inflection. The preceding branch is strong and single, but the following branch is split into two, which cross where their curvature is greatest, at some distance from the center of the spiral, and unite again at their extremities. This appearance in the components of the following branch, and the fact that the ends of both branches curve around so as to approach the center more closely than do the intermediate parts, are doubtless effects of projection, the plane of the spiral lying obliquely to the line of sight. Proceeding from the following branch are also numerous streams of faint nebulosity, which very likely may not appear in the reproduction.

In the center of the space which is nearly enclosed by the preceding branch is the fifteenth magnitude star drawn by Lord Rosse. The greater part of this space is filled with faint nebulosity, but a narrow dark bay extends inward from the opening on the north, and the star is situated in the middle of the head of this bay. The dark space and the faint nebulosity which

borders it are concentrically disposed with respect to the star.[1] It would be of great interest to know whether this singular position of the star is accidental or whether the star and the nebula are physically connected; and if so, in what way the star was left in its present position during the process of contraction. On the first of these questions an investigation of the spectrum, which will be made in due time, may throw some light. Assuming, for the present, that the star is physically connected with the nebula, it seems to me possible that the proximity of this star may account for the unsymmetrical appearance of the spiral which may be due to an actual difference in the dimensions of the two branches, or to their lying in differently inclined planes.

A comparison of the photograph with the figures in the plate illustrates in a very interesting and instructive manner, the personality of the draftsman which is commented upon by Tempel, and from which his own drawing is by no means exempt.

The most obvious tendency of the draughtsman, which is seen in this and in similar comparisons, is to prolong a line or curve beyond the point at which it actually stops. Thus all the observers of *H I, 55 Pegasi*, regarding the central part of the spiral as a straight, elongated nebula, imagined that they saw it extended in both directions beyond its real limits, and consequently represented it as having the spindle-shaped outline shown in the figures. The preceding branch of the spiral was recognized in its true character by Lord Rosse alone. The following branch can hardly be detected visually. It is barely visible with the Crossley reflector on a fine night, and its spiral structure would certainly not be suspected.

There is also a natural tendency in drawing to emphasize the details caught by the eye, while others, which are missed, may be nearly or quite as prominent. Thus the drawing and the photograph differ.

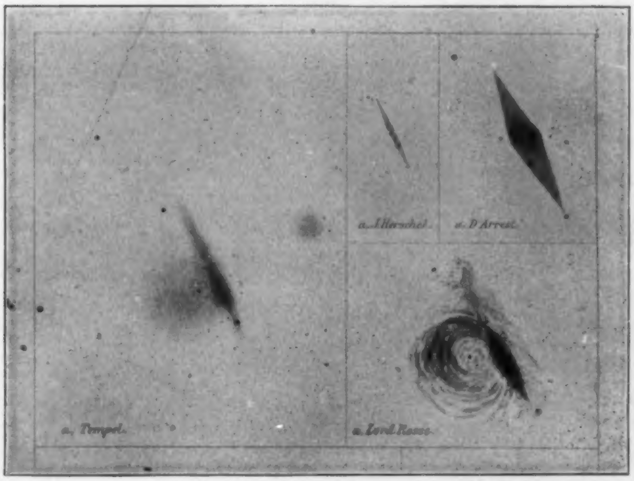
The faint nebula drawn by Tempel a little east of *H I, 55 Pegasi* does not appear on the Crossley photograph, although it

[1] It is quite likely that these details also will fail to appear in the reproduction. The largest star of the drawings, south of the nebula, is just outside the limit of the photograph.

PLATE II.

PHOTOGRAPH OF THE NEBULA *H I. 55 PEGASI*
Made with the Crossley Reflector

PLATE III.



DRAWINGS OF THE NEBULA *H' I. 55 PEGASI*

is hardly necessary to say that with an exposure of three and one half hours this instrument will photograph nebulae beyond the reach of any visual telescope whatever. Near the place of the supposed nebula there are several small stars, whose combined effect may possibly have produced an appearance of nebulosity, though they are all below the sixteenth magnitude. In the case of Lord Rosse's drawing it is probable that the numerous streaks and patches represent a manner of drawing, rather than an attempt to depict extremely minute details.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1899.

ON A NEW SYSTEM FOR SPECTRAL PHOTOMETRIC WORK.

By D. B. BRACE.

IN the comparison of intensities of the spectra of different radiants or of the different parts of the same spectrum, the greatest sensibility of the eye is attained when the determination between these elements in the field of view is the sharpest, so that in a setting for a match the boundary can be made to vanish abruptly and completely, thus giving a perfect uniformity in the intensity of the field. This latter must also be sufficiently bright to give to the eye the most favorable conditions in such a comparison. It must also be possible to measure, to at least as high a degree of accuracy, in a simple way, the variations in the intensities by which this is attained and maintain constant sources, which may be readily reproduced for comparison.

In many of the instruments these three conditions are not realized. In some we find the means for measuring a variation in intensity very accurately attained, but at the expense of the amount of the original light, so that an accurate setting cannot be obtained on account of the weakness of the field of view. When sufficient light is available, a dark line or (bright line) is usually visible between the two parts of the field, which seriously reduces the sensibility of the eye in determining a match.

In the instruments devised by Govi, Vierordt, Crova, and others, the two spectra are made contiguous by means of double slits or total reflecting prisms which, if they do not overlap, give a line of separation of perceptible width and of less intensity than the adjacent spectra or, if they do overlap, a region of variable intensity of finite width which confuses the eye in determining a match. In the polarizing instruments where double image prisms are used to affect the same thing, this gradual transition from the true intensity of one spectrum to that of the other is of finite extent. In addition to this difficulty, the

brightness of the field is reduced too much to make accurate comparisons of radiants of low intensity, as in the types used by Glan, Crova, Glazebrook, König, and others. In the instruments of Cornu and Kundt where the variation in intensity is attained by diaphragming the objective sectorially, the two spectra must be brought together by systems having the above inherent defects, which, with the liability to irregular distribution of intensities over the objective and consequent error of measurement, render them open to serious objection.

Photometers like those of Wild, Trannin, and others, which depend upon the vanishing of the interference bands of two distinct sources when properly superposed, after a method used by Babinet, are not capable of great sensibility and are decidedly objectionable on account of the severe strain upon the eye in determining the exact vanishment of the bands. The accuracy of measurement of the intensities is very high, far more so than the sensibility of the eye in making a match. The degree of accuracy to which an instrument is graduated for measuring changes of intensities should not be mistaken for the accuracy with which the eye can make the comparisons as seems to have been done in the case of one of these latter instruments and generally quoted by authorities of high experience. The constructors of such an instrument have readily obtained a graduation accurate to one tenth per cent. but have scarcely attained a setting with the eye accurate to one per cent., so that they have replaced it by several instruments which give superior results. It is doubtful if a true sensibility of the value so generally quoted for interference photometers has ever been attained to even one tenth part. It is only in one or two later types of spectrophotometers, that the sensibility in the settings by the eye have reached or exceeded the means of measuring the intensities as well as of maintaining their uniformity, notwithstanding the various types of radiants proposed as photometric standards. While much attention has been given to this latter question no radiant, with possibly the exception of the Reichsanstalt platinum standard, has been proposed which will maintain its constancy

throughout the spectrum, for an extended period of time, to within one fourth of one per cent.,—the sensibility now obtainable by the eye with a simple prismatic viewing screen. If we consider the complexity of the optical systems in some of the spectral photometers, particularly in the polarizing types, the impossibility of collating the results of different observers with one another or of obtaining absolute spectrophotometric data accurate to within one per cent. is apparent.

In the photometer cube of Lummer and Brodhun[1] we have the conditions of great accuracy in comparisons, since here the boundary between the fields compared is very sharp and vanishes for equal intensities. They have adopted different means for obtaining this condition, two of which are now in use. The first, which is more available for white light, is attained by etching sharply away a part of the surface forming the diagonal of a cube consisting of two prisms, so that total reflection from the diagonal face of the second prism abruptly terminates at the boundary of the etching, thus giving the required condition. In the second, which is suitable for spectral comparison, the prisms of the cube are cemented together after a part of one of the diagonal surfaces has been silvered to a sharp limit, so that internal reflection abruptly ceases at this line. An analogous method proposed by the writer, for obtaining abrupt transitions from several successively parallel reflecting surfaces has been used by Doubt[2] in making comparisons of color mixtures.

In order to avoid diffraction, which would destroy the sharpness of the boundary, the line of delimitation must be chosen so that it is parallel to the dispersion at each point. It is found by so doing that all diffracting phenomena disappear for a match.

In these several screens no dispersion takes place. This is effected by the usual dispersing prism which forms a part of the optical train. In the photometer here described the dispersion takes place within the prismatic viewing screen itself, thus simplifying the optical system and reducing the number of refracting

[1] *Zeitschrift für Instrumentenkunde*, February 1892; April 1892.

[2] T. E. Doubt, *Phil. Mag.*, 46, 216, 1898.

surfaces and the number of adjustments necessary to eliminate diffraction and obtain a perfectly sharp boundary.

The compound prism, Fig. 1, is essentially an equilateral prism bisected by a plane *A D* through one of its refracting edges, thus making the vertical angles at *A* of each prism equal. The face *A D* of one of the prisms, *A D C* say, is then silvered, and this silvering carefully removed, all except a strip whose length is normal to the bisected edge and whose height is about one third the field of view. The edges of the strip *S S* must be perfectly sharp and regular. This is generally effected by carefully treating with acid and finishing the bounding lines with a sharp ivory chisel. The vertical angles *B* and *C* should be the same. The two prisms are then placed together with some cement or liquid between the common surfaces *A D*. For prisms of low dispersion balsam is used, and α-monobromonapthaline in a prism of high refractive index. This latter substance is very constant under such conditions and does not attack the silver and is further colorless and very transparent to the blue and violet rays. Both substances however must be cemented in to prevent evaporation, the latter especially being quite volatile and gradually disappearing at the edges, thus contracting the field of view. If gelatine, which is not attacked by it, is placed over the edges, the evaporation can be checked and the prism made permanent. In this arrangement the prism can readily be taken apart and cleaned and refilled with new fluid, if necessary, without injuring the silver strip. On account of the slight thickness of this strip and the difference in the refractive indices between the cementing fluid and the glass, some light may be reflected from other portions of the interface than *S S* which, when this difference is marked, will produce the phenomenon of interference bands for thin films. Under certain conditions

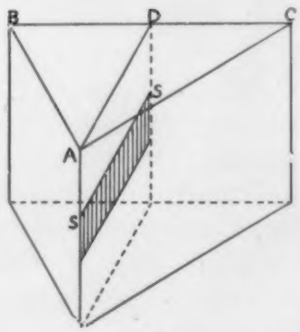


FIG. 1

these can be seen, but, as will be shown later, they disappear entirely when the eye observes directly the interface and a match is made. In general, prisms of high dispersive power are preferable especially when sufficient light is available, in order that comparisons may be made with the field as nearly monochromatic as possible. For low intensities, such as in comparisons of stellar spectra, crown glass may be advantageous. The glass selected for higher dispersion is that from the factory of Messrs. Schott and Co., Jena, Dense Silicate Flint O 102 $n_d = 1.6527$, $\left(\frac{\Delta n}{n_d - 1}\right)_{c-f} = 0.01950$. The corresponding index of a-monobromonapthaline is $n_d = 1.6582$ @ 20° C. This combination gives no appreciable internal reflection or interference and a monochromatic field of great uniformity when viewed directly by the eye through a slit of quite sufficient dimensions to obtain the proper intensities for comparisons. The above fluid is now much used for optical purposes and is readily obtained. When this is not at hand Canada balsam may be used with this glass, although interference bands are generally present when viewed with the eyepiece, but vanish when viewed directly with the eye. The results however are not as satisfactory as when the indices are nearly the same, and it is difficult to separate the prisms after a considerable length of time without destroying the silver strip. Crown-glass prisms cemented with balsam give less dispersion but greater spectral intensities and a fairly monochromatic field, particularly in the upper portions of the spectrum. For low intensities these may be used to advantage, for example, in comparing stellar spectra of the fainter stars.

Heretofore in making spectral comparisons of two sources sent from different directions through the same dispersing prism into the field of view,—notably in the well-known color instrument of von Helmholtz,—the adjacent spectra extend in opposite directions across the field. On account of this change in direction of the color gradient a perceptible change in tint at the extremities occurs, while at the same time a black vertical line is always seen separating the two halves of the field. From the

well-known optical principle that an odd number of reflections produces a reversion and an even number a rectification of the image, it will be seen that in the combination here considered we have one reflection and a direct transmission giving the same color gradient to the spectra of the two sources.

In Fig. 2, *A B C* is the compound prism, *T* and *T′* are the collimating telescopes and *R* is the observing telescope. A ray *b*, for example, from the slit of *T* is refracted at the face *B D* after which each color component meets the interface *A D* at $d \dots d'$, those rays meeting the silvered strip being reflected out through the prism in the direction $a_d' \dots a_h'$, while the remainder pass on through the prism into

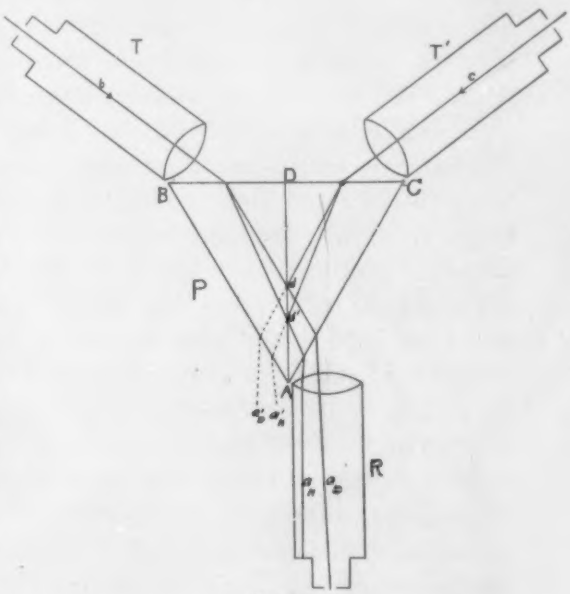


FIG. 2

the observing telescope *R* in the direction $a_d \dots a_h$, certain color components passing through the ocular slit into the eye, which sees, consequently, the interface *A D* illuminated with this color, excepting the portion of the field covered by the silvered strip. Similarly the collimating telescope *T′* is so adjusted that a ray *c* from its slit is refracted at the surface *D C* at such an angle that its corresponding color components meet the interface at $d \dots d'$, some passing on through the prism and emerging in the direction $a_d' \dots a_h'$, while those meeting the silvered strip are reflected on out of the prism into the observing telescope along the direction $a_d \dots a_h$, like color components to the first ray *b* thus passing

through the slit R to the eye, which sees illuminated, with the same color, the part of the field covered by the silvered strip, the boundary between the two portions being thus abrupt and sharply defined. On varying the intensity of the components this is made to vanish completely, so that the eye sees the interface $A\ D$ illuminated uniformly with the same color and without any perceptible light or dark lines. The field may be stopped down to a suitable size by a circular diaphragm on R. If now the telescope R be shifted, the field appears illuminated successively with all the different colors of the spectrum.

Since it would introduce mechanical complications, and the error in color intensity would be inappreciable, if observations were made for minimum deviation in every case, the prism is adjusted for the mean color of the direct ray b. This is effected by inserting the eyepiece in R and observing with the cross hairs the sodium line from the slit of T. The compound prism need have but slight movement and is to be permanently fastened to its base. The collimating telescope T' is then adjusted for the same line and then clamped. In order that the different color components of the same original ray may pass through the same point in the telescope R, its axis of rotation should be about the radiant axis of color C, while its optical axis should intersect the prism one fourth the distance from A to C. A like condition applies to the collimating telescopes T and T'. For most purposes the collimators may be permanently fixed, the telescope only being mounted for rotation. The distance from the slit of R to the interface should be approximately 25 cm,—that of distinct vision. In the adjustment with the eyepiece, secondary images or ghosts are seen. These are produced by interference of residual reflections at the interface when the cementing fluid is of a different index from the glass. When the glass and the cementing fluid have the corresponding indices stated above the difference between the internal losses of the two sources arising from these and other causes is small. When the cementing fluid is removed and only the surface of the prism which contains the silver strip illuminated by the collimator T', the two parts of

the field lighted by the strip and by total reflection appear approximately of the same intensity, showing that the reflection from silver in this way is approximately total for all colors.

Dispersing prisms of higher indices may be used, but the internal absorption becomes marked. Furthermore experience has shown that, with denser glass, surface oxidization takes place after a time, thus changing the constant of absorption for different colors, a factor which must be invariable when the absolute color gradient is being determined.

It is evident at once that this construction may serve, with the use of an eyepiece, with one collimator, as a simple, and with two collimators, as a comparison spectroscope. On the other hand, from the symmetry of the optical system, a single prism spectroscope or a spectrometer may be immediately transformed into a spectral photometer by replacing the ordinary prism with such a compound prism and adding a second collimating telescope.

This type of direct viewing spectrophotometer is especially suited for making comparisons of very narrow or line spectra, such as stellar spectra, lunar or solar spectra of very limited areas, and of spectra from point or line radiants, etc. In color comparisons, a finite field of view must be presented to the eye in order to make accurate comparisons. This may be accomplished by placing a diffusing screen in the focal plane to receive the spectra, in which case there is a very great loss of intensity, only a small fraction of the rays reaching the eyes. It may also be obtained by using a high power eyepiece or a cylindrical lens to broaden the spectra, but serious distortions and consequent variation in uniformity of the field result.

Fig. 3 shows an instrument attached to a large telescope for spectrophotometric observations of the heavenly bodies. It is evident that all the light of any color gathered by the astronomical objective from one point reaches the retina; and the narrowest line spectra, when observed directly by the eye through the slit of the viewing telescope of the photometer, produce a full field of perfect uniformity. On removing the

collimator slits and sending the light from two stars directly through the respective collimators, comparisons of their spectra can at once be made with a field of view of sufficient aperture to give the eye the conditions of greatest sensibility. Observations on the spectrum of Capella with a 4-inch telescope and its attached single prism spectroscope with the slit thrown wide and the eyepiece removed showed a field of more than sufficient intensity to make good comparisons; from which it is estimated that accurate stellar spectral measurements can be made down to the fifth magnitude with a 40-inch objective. As will be referred to later, comparisons have been made with intensities representing a range from one to two millions. On examination, the Moon also showed abundant light to determine the craterous luminosity gradient in spectral elements. Atmospheric absorption may also be determined for specific parts of the spectra of different luminous bodies.

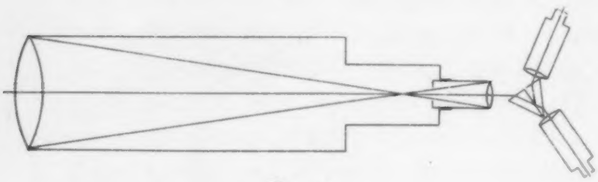


FIG. 3.

Fig. 4 (Plate IV) shows a spectrophotometer as constructed by Schmidt & Haensch, of Berlin, for physical purposes, one ninth actual size. The collimator T is fastened to the body of the instrument. The collimator T' and the telescope R are mounted on separate radial arms so that each may be displaced independently about the axis of the instrument. Each arm is mounted with a micrometer screw and clamp which engages the ring Q so that any setting may be attained and the finer adjustments made with the micrometer screws N and N' respectively. M is a vernier and scale for determining the position of R and the part of the spectrum observed. The collimator T may be mounted with a unilateral slit, but the slit of T' should be bilateral and graduated to at least one two-hundredths of a millimeter. Both slits are mounted with total reflecting prisms for comparison spectral work, and for photometric work with the eyepiece after the method of Vierordt. The total aperture of

PLATE IV.

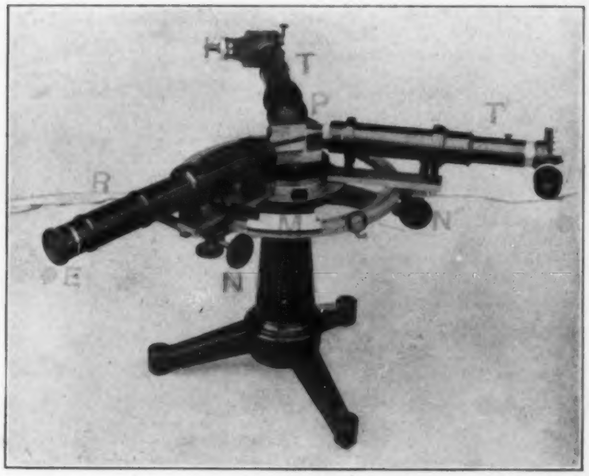


FIG. 4.

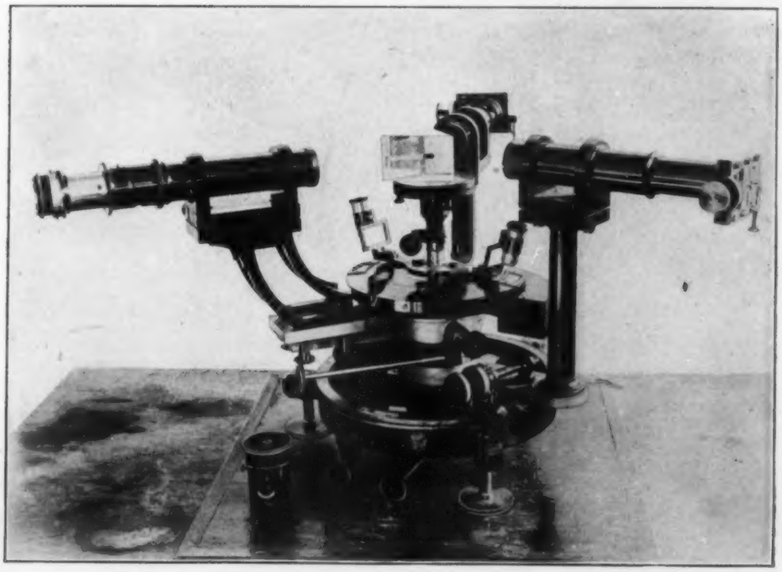


FIG. 5.

the slits need not exceed two millimeters. The telescope *R* is provided with an eyepiece *E* which fits over a double adjustable slit in its focal plane and from which it can be readily removed and the eye applied directly. The compound prism *P* within which may be seen the silver strip, is mounted above the axis of the instrument on a plate which can be shifted laterally and then screwed to the instrument when the minimum deviation has been obtained. The prism and telescope objectives are carefully housed to avoid diffused light which may interfere seriously in delicate comparisons. Special care must be exercised to eliminate extraneous light. The silver next the glass upon which it is deposited is used as the reflecting surface since its outside face is subject to oxidization and is not as perfect. This strip is upon the prism next the collimating telescope T', carrying the bilateral slit. With this instrument, the setting for minimum deviation for sodium light is effected with the collimator *T* first, using the compound slit in the telescope or cross hairs in making the setting. *P* is then fastened permanently and *R* clamped; and T' then adjusted for minimum deviation and clamped. On removing *E* and illuminating both slits with the same amount of light, the eye should see the field of approximately uniform color and intensity when *R* is shifted about the axis of rotation through the spectrum.

The high sensibility of the instrument is readily seen by the sharp outlining of the horizontal strip against the circular field of view when the base is jarred slightly. The least unsteadiness of the illuminants is manifested in a similar way. The arc *M* may now be graduated for the different wave lengths by inserting the eyepiece *E* and observing the Fraunhofer lines. The width of the occular slit may vary from 0.5 mm to 1 mm depending on the dispersion of the prism *P*, and the height may be somewhat greater except when line spectra are being compared with one another or with a broad spectrum. In this case it should be stopped down to the width of the line spectra. This bilateral ocular slit may be rapidly and continuously adjusted by rotation of a spirally slotted disk which engages the lateral jaws. The

vertical jaws are independent and adjusted by hand. The slit may be turned through ninety degrees for adjustment for line spectra if desired. The aperture of the telescope R is stopped down to 15 mm, its focal length is 25 cm, and the width of the strip which is adjusted to the center of the field is 5 mm.

Fig. 5 (Plate IV) shows a spectrometer after a model by von Lang, as constructed by Schmidt & Haensch, Berlin, and fitted with an extra collimator. Each collimator has a bilateral slit and comparison prism and is mounted on an independent arm which is provided with a micrometer screw for fine motion, the circular base of the model being faced down to allow independent motion and settings. The observing telescope has also been fitted with an ocular slit and a circular stop at the objective. This instrument has been found to be equally efficient with the special instrument described above. These additional accessories add about 20 per cent. to its original price. (Cf. Schmidt & Haensch, Catalogue 1896, No. 55.)

The regular spectrophotometers already described may be had for about 560 M., and with some simplifications for about 500 M.; for example, if both collimators are fixed and the objectives reduced to an aperture of 15 mm, the observing telescope alone being provided with a micrometer screw and without vernier and scale. This is sufficient for the purpose of photometric comparison of different spectra. If comparisons of adjacent elements of the same spectra are desired for the purpose of determining the curve of luminosity, one or both collimators should be provided with graduated micrometer screws. Any two adjacent elements can then be thrown into the field in succession and "step by step" comparisons be made clear across the spectrum.

In the optical system which has just been described almost all the light from the slit reaches the retina and in a way to give the highest sensibility for comparison, so that the first and second conditions premised for photometric work have been realized. The third and fourth conditions, namely of measuring the intensities and of obtaining constant and reproducible sources have in part been realized by simple methods.

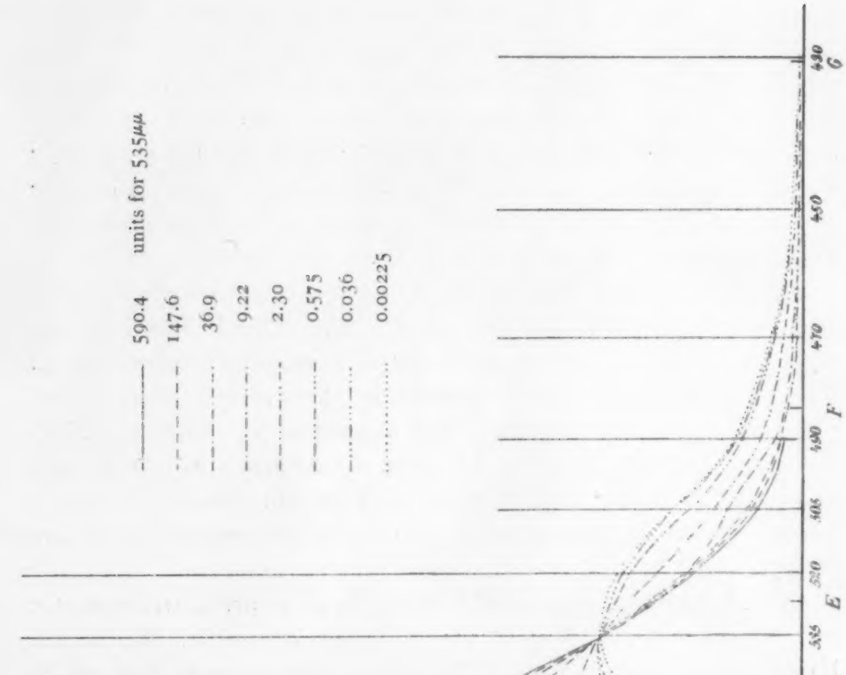
By the use of Nicol prisms, double image prisms, or other polarizing systems, small changes of intensity are very accurately determined. However, these introduce errors for depolarization and they also reduce the amount of light which reaches the retina so much (at least one half) that accurate readings cannot be made for low intensities. The direct variation of the distance from the slit or from a screen, is too cumbersome and inconvenient for permanent spectrophotometric work. The rotating sector, as used first by Talbot and latterly by Lummer and Brodhun at the Reichsanstalt, is perhaps the most reliable method when properly applied, but it is a tedious and slow process for permanent work. The method of Vierordt of measuring the width of a unilateral or a bilateral slit introduces errors of several per cent. in different parts of the spectrum for even a bilateral slit, as has been shown by Lummer and Murphy.[1] That this should be so is made evident by inspection of the curve of luminosity for any source.

Fig. 6 represents curves of luminosity of a normal trichromatic eye obtained by A. König[2] for different intensities, the source being a triple gas burner. The unit of intensity is that which the eye with a 1 sq. mm diphragm receives from a white screen of magnesium oxide illuminated by a 0.1 sq. cm of melting platinum (=1.7 candle power) at a distance of 1 meter, the intensity of the comparison color, $535\mu\mu$, being given in the diagram for the corresponding curves. In general, where the slope of the curve is not a straight line, the light entering the slit will not be proportioned to its width. For a unilateral slit it will only be true when the curve is horizontal. For a bilateral slit it will be only true for the parts of the curve which are straight lines. This error may be large, depending on the width of the slit and the part of the spectrum examined.

The further difficulty of using the slit-width, especially for narrow widths, is in determining the true zero. Errors in the screen also enter, as well as those due to selective absorption in

[1] This JOURNAL, **6**, 1.

[2] *Beiträge zur Psychologie und Physiologie der Sinnesorgane*, 1891.



590.4 units for 535μμ
147.6
36.9
9.22
2.30
0.575
0.036
0.00225
3,8
3,6
3,4
3,2
3,-
2,8
2,6
2,4
2,2
2,-
1,8
1,6
1,4
1,2
1,-
0,8
0,6
0,4
0,2
670
650
625
605
590
575
555
535
520
505
490
470
450
430
B
C
D
E
F
G

FIG. 6.

the optical system of the instrument itself. All of the errors, however, may be eliminated by the direct optical calibration of one of the slits, a simple and exact method of the slit readings not heretofore used. With this calibration, observations can be made rapidly and as accurately as with the direct use of polarizing prisms or of the variable rotating sector. Both of these methods require elaborate and expensive accessories, while the latter is a tedious although accurate process. The means of calibration adopted as the most available and accurate is the rotating sector, which Lummer and Brodhun have shown to be the most reliable method. The sector should of course be driven considerably beyond the speed of perceptible flickering. Under these conditions the effect on the retina over sufficient ranges in intensity is simply proportional to the integral amount of light which reaches it. It is then only necessary to compensate for any known change of intensity by varying the slit-width to obtain its true optical value. To accomplish this, a simple cardboard disk, Fig. 7, is divided into any number of equal parts, for instance eight, and mounted on a whirling table or motor before one of the collimators of the spectrophotometer. This may be adjusted so as to bring any one of the sectors in front of the slit so that the intensity may be made to vary from one down to one eighth successively. If the slit to be standardized is mounted with T', Fig. 2, the sector may either be placed before it or before the slit of T. Having first obtained a match, the disk is set into rotation and the slit of T' adjusted again for a match. The slit reading gives the relative optical value corresponding to the sector used. For example, if the sector was half of the circumference, the slit reading is for one half its optical value if the rotating sector be before T, and twice its value if it be before T'. Successive values may thus be obtained dividing up the slit into proportional parts. The optical value for any other

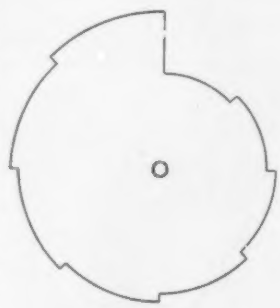


FIG. 7.

width or reading of the screw may be obtained, by interpolation, to any desired degree of accuracy. This calibration evidently eliminates all instrumental errors. This process may be repeated for all colors by shifting the telescope *R*. Of course, either of the methods of measurement may be used for calibrating and then dispensed with. Once the calibration curves are obtained the slit should never be closed and the setting should always be made for one direction of the screw.

The curve of luminosity depends on the nature of the source and on subjective conditions. A definite distribution of the radiant energy will produce a certain curve for any one eye. If each element be changed in the same ratio, the luminosity curve will not vary in the same way. This influence of the absolute intensities on the color gradient was first observed by Purkinje. The retinae of different eyes show a variation in the degree of sensation produced, not only for the same amount of energy for any one color but also different relative sensibilities for the different spectral elements. Thus trichromatic eyes may show greater discrepancies in their curves of luminosity than exist between one of them and a dichromatic (red, green or blue color blind) eye. A monochromatic eye will have its own curve of luminosity which will depend on the relative distribution of energy and its value for any one part of the spectrum.

Fig. 6, already referred to, illustrates these variations for a normal trichromatic eye, each ordinate being multiplied by a factor which makes the ordinates of all curves the same for the wave-length 535 $\mu\mu$.

Fig. 8 shows the curve of different eyes for the same distribution of energy when the intensity is high, and also these curves reduced to a common ordinate for wave-length 535 $\mu\mu$. The curve of a normal trichromatic eye for a uniform high distribution of energy throughout the spectrum is also given.

Fig. 9 gives the corresponding curves for very low intensities. These curves represent the changes produced by varying the intensity of the original source,—a triple gas burner. Other sources, such as incandescent, filaments, molten platinum, the arc

light, would give their own characteristic curve under these conditions. For low intensities the slope of the curves of different eyes are approximately the same and the calibration of the slit could be used by different persons without serious error, but for

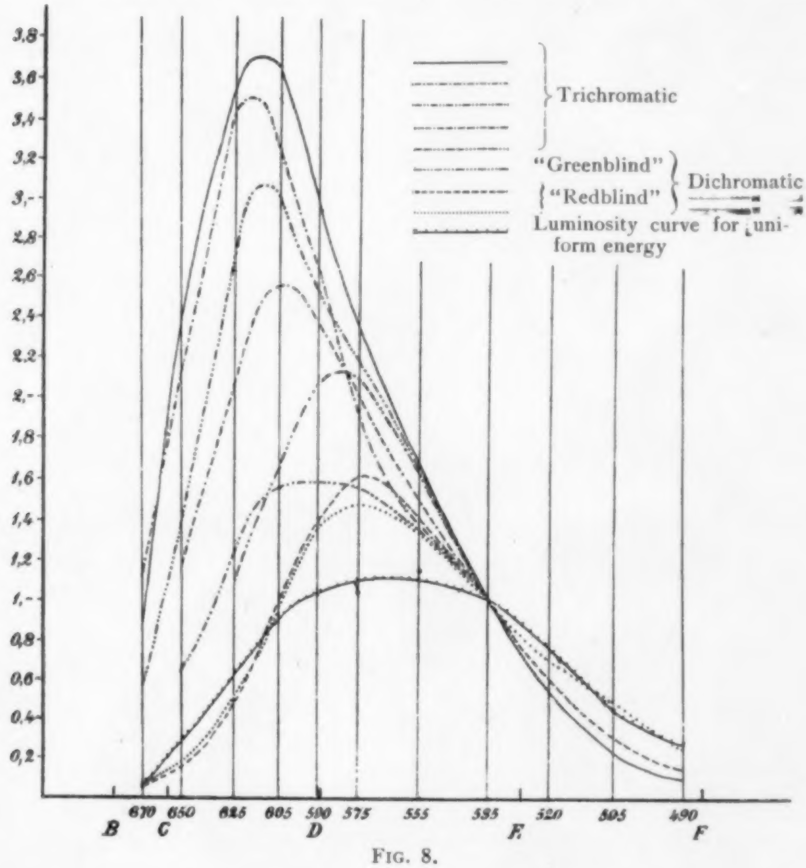
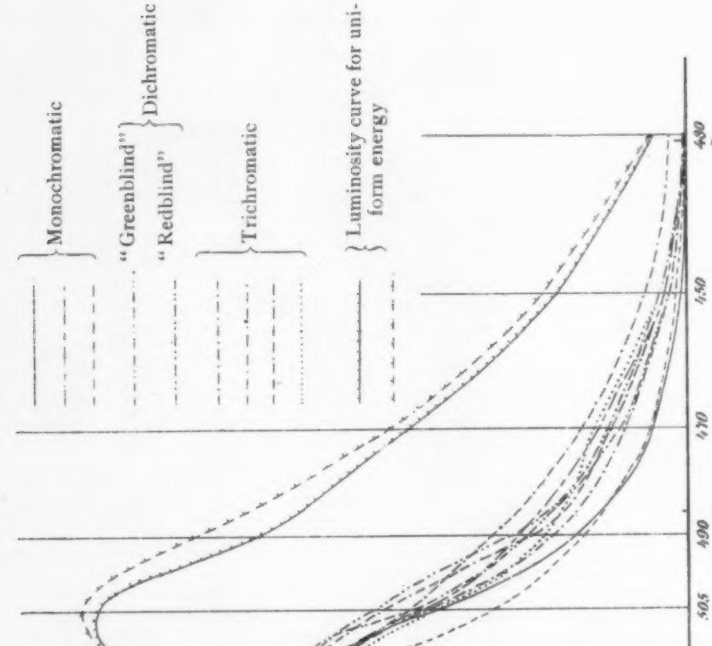


FIG. 8.

other intensities, the calibration should be made for each eye. This error becomes less and less as the width of the slit is reduced and the dispersion increased. With the flint glass recommended and a source readily available, the slit-width of 0.25 mm practically eliminates this error and gives sufficient light for the most sensitive comparisons in general work.



Monochromatic
"Greenblind"
"Redblind"
Dichromatic
Trichromatic
Luminosity curve for uniform energy
1,6
1,5
1,4
1,3
1,2
1,1
1,-
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
B
670
C
650
625
605
D
590
575
555
535
E
520
505
490
F
470
450
430
G

FIG. 9.

Experiments upon the eye indicate that its curve does not change over long periods at a time; hence if the source before the calibrated slit can be maintained uniform for such a period, the conditions of the problem are met. The only sources giving sufficient constancy have been found to be incandescent filaments maintained and regulated by electric currents. Filaments of carbon give approximately the same curve over considerable variations in voltage for slit-widths of 1 mm or less. These are usually placed in frosted globes (preferably in close spirals and near the glass). To obtain areas of uniform intensities additional ground glass plates are used when necessary. Under favorable conditions the instrument has given for the mean colors of the spectrum a variation of only 0.5 per cent. from a mean of ten settings, or a mean error of 0.25 per cent. for one setting. Obvious applications of the optical principle in this instrument may be made to other instruments of comparison, such as those for comparing mixtures of spectral elements.

Instead of using the slit as above outlined, incandescent line filaments at the focus may be used directly instead. These may be vertical, and the intensities determined for the different colors by previous comparison and "optical" calibrations of the rheostats in series with the filaments. Much greater intensity can be obtained in this way, and a perfectly uniform field is assured. For measurements of high absorption, this method may be utilized if the absorbing system can be interposed between the objective and the line radiant at the focus of the objective without disturbing the optical definition.

Good results have also been obtained by cutting down the height of the ocular slit and placing the filament horizontal and close to the collimator slit. By varying the resistance of the circuit, a match is obtained without using the readings of the screw. In these systems the errors to which the slit is subject are entirely avoided. The horizontal filament may also be maintained at a constant voltage and readings made with the slit. In this way a uniform field of high intensity is obtained, since when the globe is placed at a distance, the slit acts like a lens

to form an image of the filament within the compound prism or viewing screen which the eye sees directly, and any lack of uniformity becomes evident, so that additional ground glass plates are required, thus materially reducing the intensity.

For measurements where a standard of comparison is desired an incandescent platinum wire is to be occasionally substituted for one of the slits (or filaments) and checked up with the standardized slit and its radiant (or standardized filament). The platinum wire is to be of a definite section and its temperature is to be determined after the manner of the Reichsanstalt platinum photometric standard by making the ratio of the direct radiations to those transmitted by a quartz cell of water 1 cm thick some chosen value, for instance ten.

Physical Laboratory,
University of Nebraska,
December 1899.

CALIBRATION OF THE SLIT IN SPECTRAL PHOTOMETRIC MEASUREMENTS.

By E. V. CAPPS.[1]

THE advantage of the slit in spectrophotometric measurements is the great rapidity with which observations can be taken, but, for its use, it becomes necessary to obtain reliable calibration.

The contents of this paper are the results of preliminary experiments which were made in connection with the study of absorption of light in aqueous solutions.

The intensity of the light at a given point in a spectrum formed from a slit of finite width, is the resultant of all the wave-lengths lying in a given space of a pure spectrum, so that it is determined by the width of the slit. By increasing the slit-width, light of greater or less wave-lengths, depending upon the direction of motion, is superimposed upon the original amount, and thus the increase of intensity will depend upon the ratio of the mean intensity of the wave-lengths added, to the mean intensity of the original wave-lengths.

The curve of luminosity of the spectrum is approximately represented in Fig. 1, the exact form depending upon the source of light.

It has been shown by D. W. Murphy (this JOURNAL, **6**, 1) by deduction from the energy curve of the spectrum and by experiment, that the intensity of the light which is received by the eye does not vary directly with the width of the slit. It follows, therefore, that with an unilateral slit the intensity would conform to the direct ratio law only where the curve is parallel to the axis. With a bilateral slit the direct ratio would hold at all points where the curve is a straight line, as the points *a*, *c*, *d*, and *f*. It also follows from the curve of luminosity that where the curve is convex to the axis as at *ab* and *ef*, the intensity of the

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light increases in a greater ratio than the width of the slit, the reverse being true for the concave portion *b, c*.

The experiments of Murphy were performed with a Lummer-Brodhun spectrophotometer. A revolving sector was used to weaken the light from one source. The sector was set to decrease the intensity of the light to one half of the original amount, and this was compared with the ratio of the corresponding slit readings. The results obtained show a variation from the direct ratio of 2 per cent. from the middle portion of the

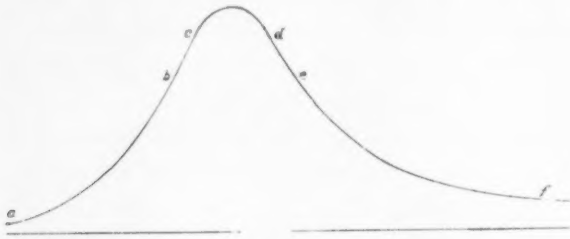

FIG. I.

spectrum and nearly 10 per cent. for the red end. They also show that the intensity increases faster for the convex portions than the width of the slit, and slower for the concave portions, thus confirming the deductions from the luminosity curve. The present experiments were performed with a spectrophotometer designed and described by Professor Brace.[1]

The plan of the instrument is similar to the one employed by Professor Brace.[2] Two collimators, and fitted with bilateral slits, and graduated to hundredths of a millimeter, are mounted upon a circular base. Opposite to these, and fitted with a circular scale is the telescope. By means of a microscope it is possible to read its angular position to ten seconds. At the eye end there is an adjustable slit which limits the vision to a particular portion of the spectrum. When used as a spectrometer, the eyepiece is attached and the slit takes the place of the cross hairs. The prism consists of two right angle prisms which form, when clamped together, an equilateral triangle.

[1] See p. 6. [2] See p. 11, Fig. 2.

The space between is filled with some liquid whose refractive index is the same as that of the glass. At the middle of one of the internal surfaces and parallel to the base there is deposited a strip of silver 5 mm wide.

The prism when in position is set for minimum deviation. The light from one source passes through the collimator and is refracted by the prism into the telescope. The light from the second source passes into the prism until it meets the silver surface, where it is reflected into the telescope with light from the first slit. By adjusting the angular position of the second slit, the spectra from the two sources are made to coincide, line with line. A cap with a circular opening is fitted over the objective of the telescope, and upon viewing the field through the eye-slit there is seen a circular patch of light, crossed by a band of the same color. The setting is made by varying the width of the slit until the band vanishes.

The greatest difficulty encountered was to obtain a constant source of light, that at the same time would give great intensity and uniformity. The ordinary incandescent lamp, operated on a storage battery circuit, fulfilled the first condition, but in order to satisfy the third, it was necessary to place the lamp at least 25 cm behind a ground-glass screen, thus greatly reducing the intensity of the light. The spectrophotometer was originally set up with a crown-glass prism, using the above described source of light. This was satisfactory with the exception that measurements could not be made in the extreme violet with the absorbing liquid before the slit. It was also observed in studying the absorption of solutions that the spectrum was not sufficiently pure to obtain a match with respect to color, if the absorption exceeded 25 per cent. To overcome this difficulty, a flint-glass prism was substituted for the crown, and this necessitated an increase of intensity of the source of light to at least fifty times. After many unsuccessful attempts the author succeeded in constructing an incandescent lamp, having a flat filament 12 mm wide. This could be placed as far as 12 cm from the slit and still cover the field. When viewed directly the

intensity was nearly one hundred times as great as it was possible to obtain by the old method.

The many difficulties of observation were entirely removed by the adoption of the flint-glass prism and these special lamps, as sources of light. The dispersion of curves of the crown and flint-glass prisms are shown in Fig. 2.

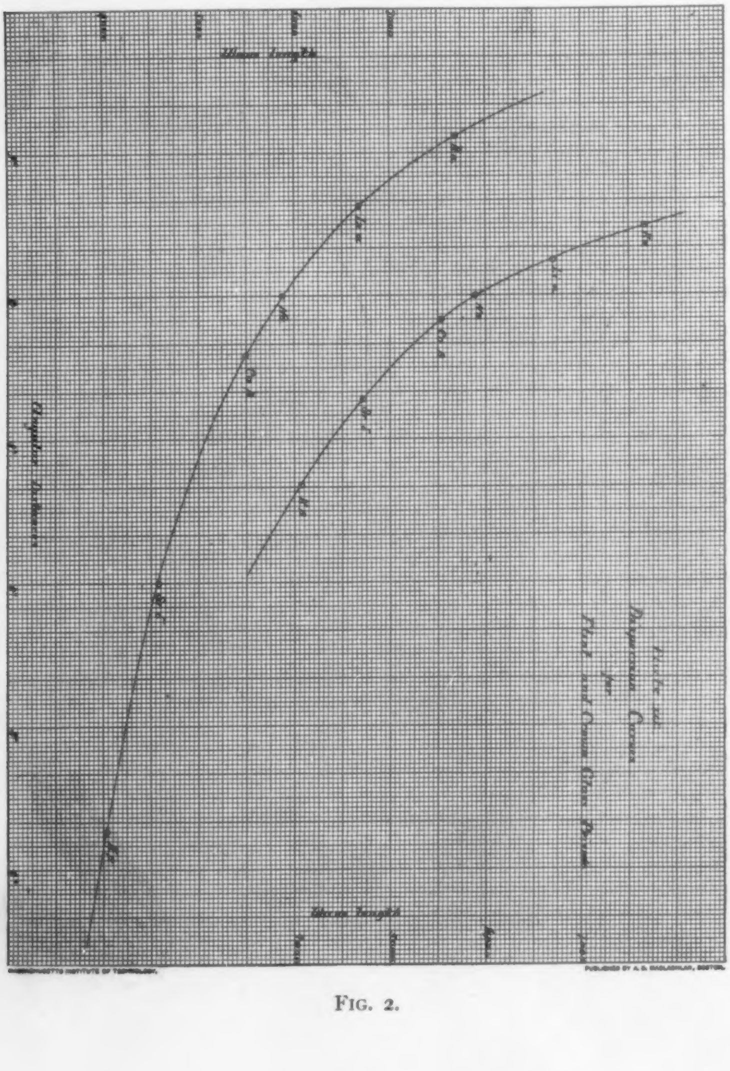

FIG. 2.

The method of making the calibration consisted in the use of a revolving disk notched at the periphery, as shown in Fig. 3, so that the intensity of the light is decreased one eighth of the whole for each notch. It was made of bristol board, and blackened with lampblack and shellac. The disk was fastened to a rotating wheel mounted upon a sliding base, which could be pushed along by a thumbscrew, the driving belt being made of spiral spring. It was driven with a small electric motor, and could be run at a high speed without vibration. The readings were taken as follows: Ten settings were made with the disk at rest. Then with the disk running, ten settings were taken for each successive notch in front of the slit, followed by ten more with the disk at rest. The mean value of the first and last ten was taken as the setting for 100 per cent. transmission, and the ratio of the settings for each notch of the disk to this value, was taken as the transmission as shown by the slit. Readings were taken for different slit-widths and different parts of the spectrum. The results give the calibration for both the flint and crown-glass prisms, subject to the conditions as previously explained.

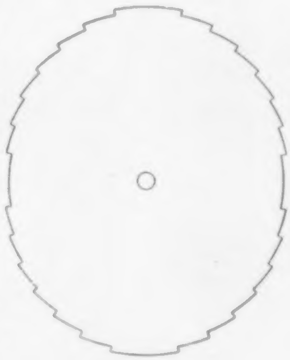

FIG. 3.

The accuracy with which the settings could be made is shown in Table I, which was chosen arbitrarily. The mean probable errors for the first column is seven tenths of 1 per cent., but the error can be reduced to one fourth of 1 per cent. under favorable conditions. The mean values of the first and last columns only vary from each other by three tenths of 1 per cent., the time interval between the two observations being thirty minutes. This shows how nearly the lights remain constant during the time interval of observation.

The results for the crown-glass prism are given in Table II, and the curves for the same in Figs. 4 and 5. The lines headed "per cent. by disk" are the transmissions as determined by the

notches of the disk. The lines headed "per cent. by slit" are the transmissions as found by the ratio of the slit readings. The curves show that for the red and blue parts of the spectrum, the intensity increases faster than the width of the slit, while the reverse is true for the yellow. This exactly confirms the

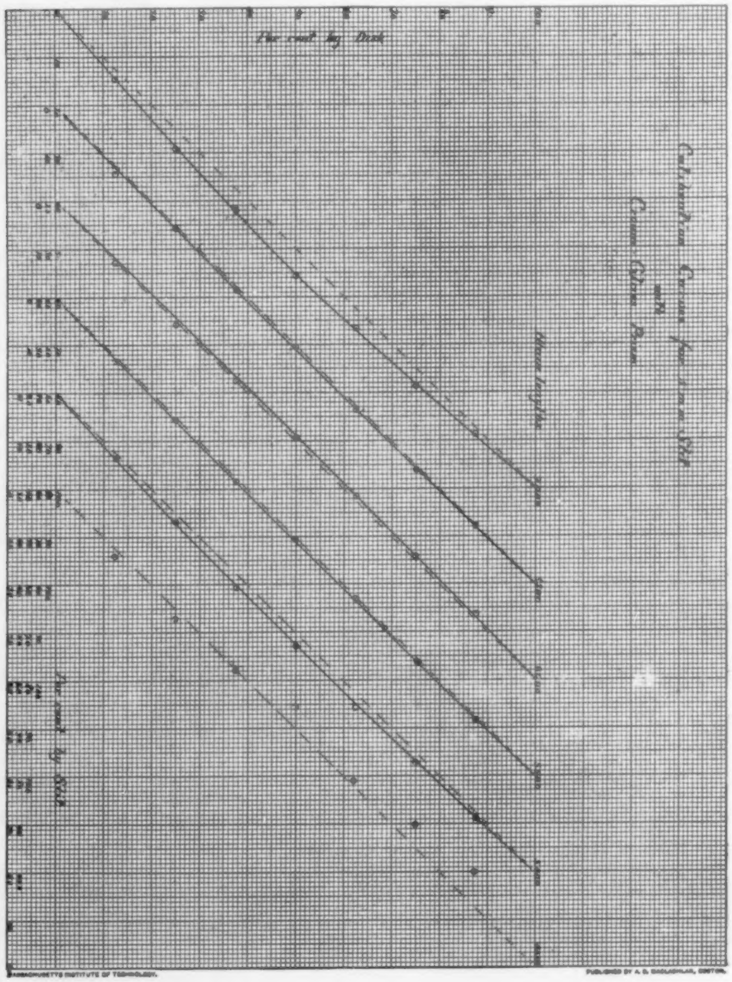

FIG. 4.

deductions made from the curve of luminosity. The effect of increasing the width of the slit is to increase the variation for all parts of the spectrum. The variation in the red for a 2 mm slit is seen to be very great.

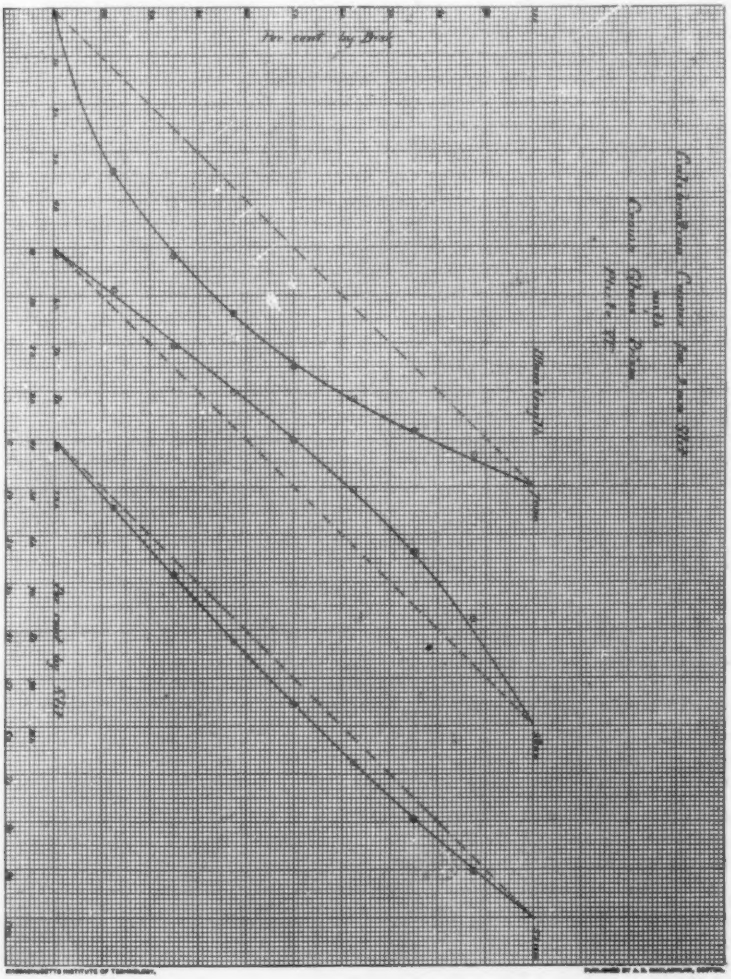

FIG. 5.

TABLE I.

CALIBRATION OF SLIT.

Slit 0.5 mm. Wave-length 0.7000 μ.

	Zero	12.5	25	37.5	50	62.5	75	87.5	Zero
Settings	.456	.026	.092	.151	.228	.275	.350	.400	.451
	.462	.028	.099	.160	.226	.286	.350	.403	.458
	.457	.028	.091	.162	.228	.278	.345	.404	.457
	.459	.029	.096	.152	.220	.280	.350	.402	.457
	.461	.030	.098	.153	.226	.280	.353	.409	.465
	.456	.029	.093	.153	.228	.2[illegible]3	.350	.400	.457
	.452	.029	.093	.152	.229	.278	.340	.403	.450
	.455	.030	.095	.152	.222	.280	.355	.409	.451
	.460	.031	.093	.155	.225	.276	.340	.404	.462
	.459	.027	.096	.161	.229	.276	.342	.405	.456
Mean	.4577	.0287	.0947	.1556	.2261	.2792	.3475	.4039	.4564

TABLE II. (Calibration for 0.5 mm slit.)

Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100	Wave length
Per cents. by slit	14.5	29.1	42.0	55.6	66.7	78.7	88.6	100	7000
	14.2	25.5	38.5	50.2	63.1	76.0	87.5	100	6500
	12.6	25.7	37.3	49.4	61.9	73.9	86.3	100	6000
	13.4	25.8	38.2	50.6	62.7	76.2	88.1	100	5500
	13.7	26.7	40.5	52.9	65.4	76.92	88.5	100	5000
	14.2	27.0	37.2	45.3	61.0	70.2	80.7	100	4500
	Slit 2 mm wide.								
	33.9	51.7	63.8	74.7	81.4	88.1	93.3	100	7000
	8.7	20.2	30.2	40.2	50.4	63.6	77.8	100	5800
	14.4	28.2	42.4	55.4	67.4	79.2	90.2	100	5200

TABLE III. (Wave-length 0.7000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit[1]	13.8	27.1	39.3	53.5	64.4	77.9	89.3	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.9	28.8	42.5	55.3	68.6	78.1	90.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	16.8	31.9	45.4	58.4	69.9	81.1	90.4	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	19.2	36.2	50.8	63.5	74.7	82.7	90.5	100

TABLE IV. (Wave-length 0.6500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.2	27.4	38.9	50.8	64.7	79.0	90.8	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.7	26.4	38.8	51.3	64.5	76.4	88.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.9	27.8	40.7	54.5	66.4	78.8	91.0	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.3	27.7	41.5	57.1	66.9	79.9	89.0	100

TABLE V. (Wave-length 0.6000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.8	26.2	38.4	50.4	62.6	74.5	86.5	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.1	25.5	37.3	49.9	62.0	74.0	86.4	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	12.5	25.0	36.9	49.3	61.0	74.9	86.9	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	12.3	23.1	35.9	47.5	59.9	72.6	86.1	100

TABLE VI. (Wave-length 0.5500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.2	26.2	38.2	50.9	63.0	75.5	87.6	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.6	26.3	39.1	51.8	63.5	75.8	87.8	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.6	26.4	38.9	52.1	63.8	76.3	87.9	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.9	26.5	39.2	52.3	64.1	77.1	88.5	100

TABLE VII. (Wave-length 0.5000 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	13.2	26.1	38.5	50.9	62.7	74.8	87.0	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.7	27.8	40.4	53.9	65.5	77.0	88.8	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.6	29.0	43.1	55.7	67.7	79.2	89.8	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	15.7	31.1	45.0	57.6	70.5	81.0	91.5	100

TABLE VIII. (Wave-length 0.4500 μ.)

Slit 0.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.8	27.2	38.7	49.1	61.3	73.4	87.1	100
Slit 1 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.6	26.5	38.9	50.7	62.3	74.7	86.5	100
Slit 1.5 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.8	28.1	41.4	53.4	64.7	77.9	89.4	100
Slit 2 mm.								
Per cent. by disk	12.5	25	37.5	50	62.5	75	87.5	100
Per cent. by slit	14.5	27.7	41.1	52.2	66.2	77.9	89.4	100

The results for the flint-glass prism are given in Tables III to VIII inclusive, the corresponding curves are plotted in Figs. 6 to 11. The general character of the results are the same as was obtained for the crown-glass prism with the exception that the variation is not nearly so large for the same slit-width and wave-length. The curves for the yellow and the violet, with a slit-width of 0.5 mm, cross the direct ratio line near its middle point. This may possibly be due to slight internal reflection, as nothing of this nature could be detected in the case of the crown glass. To test the relative constancy of the sources, one was dimmed by cutting down the current until the intensity was only one half of the other. The slit was then calibrated for the 50 per cent. notch, and the results were the same as previously obtained when

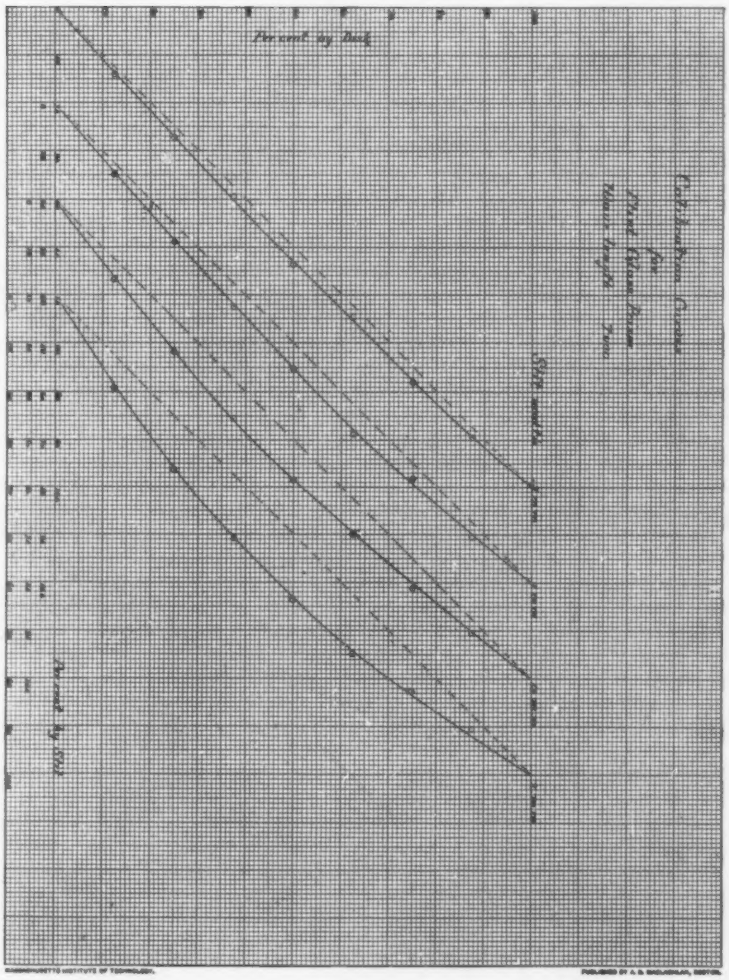

FIG. 6.

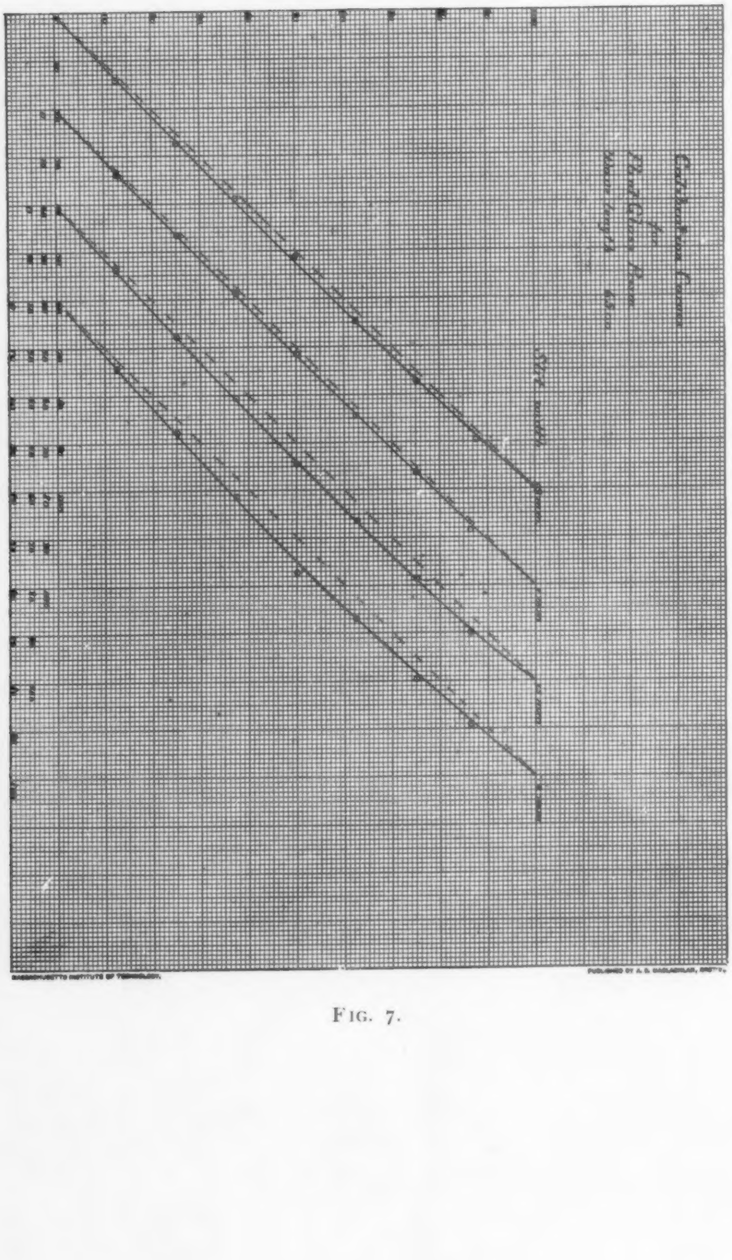

Fig. 7.

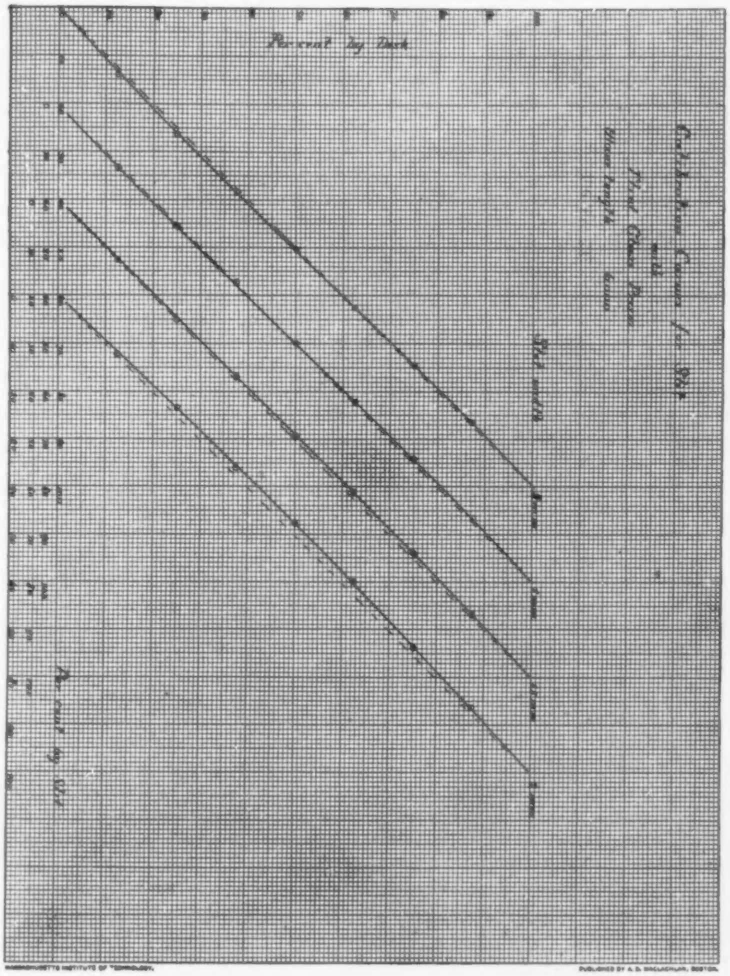

FIG. 8.

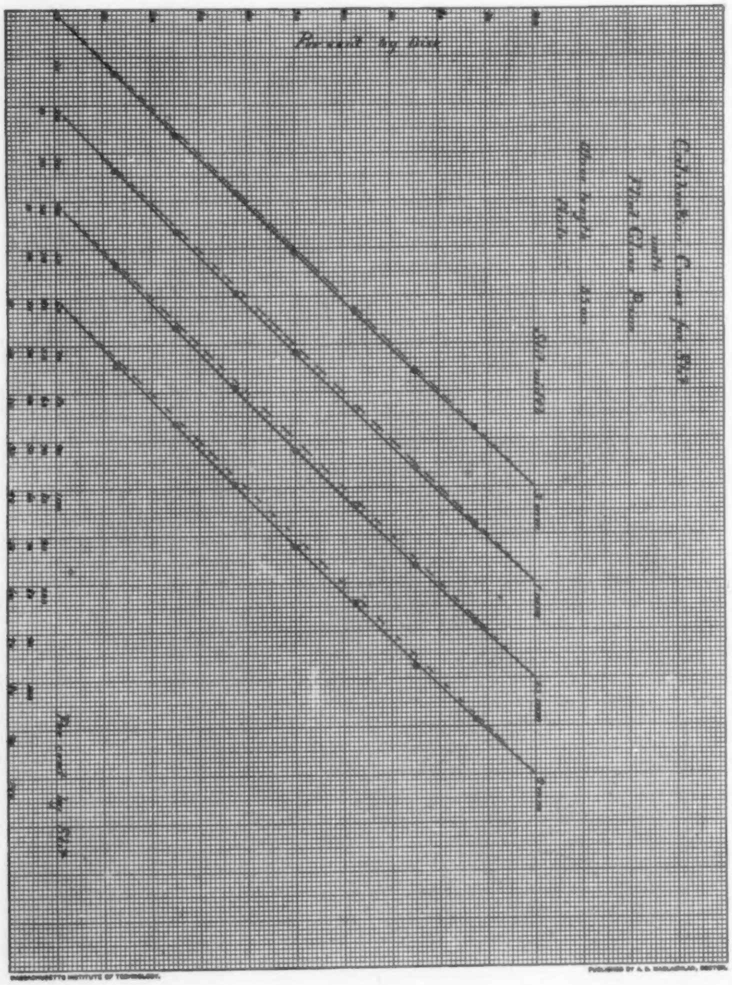

FIG. 9.

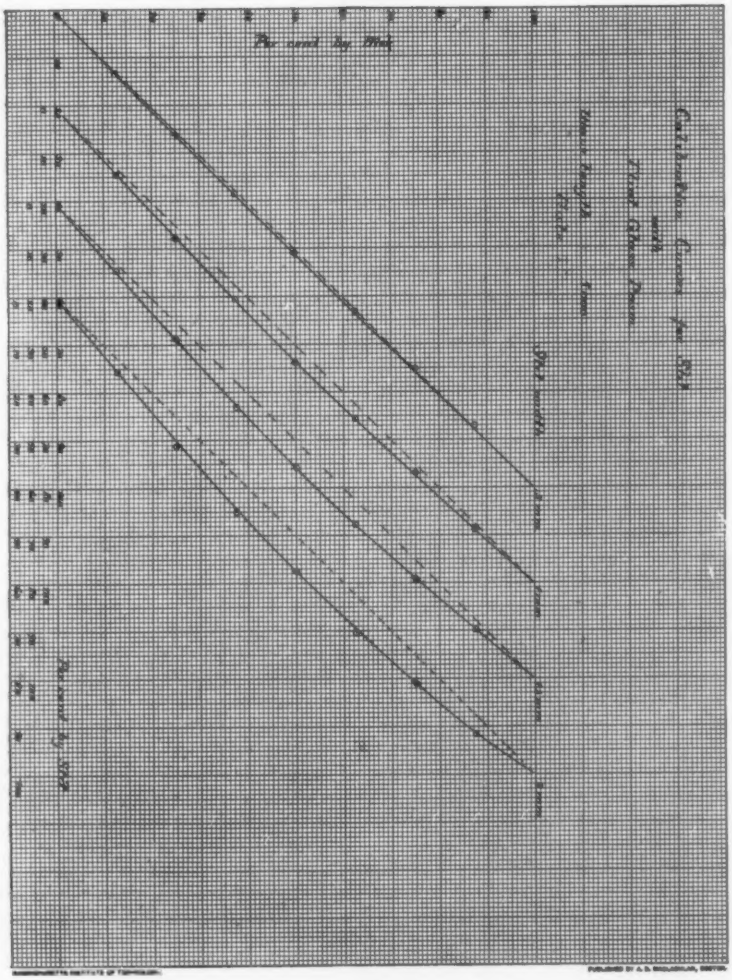

FIG. 10.

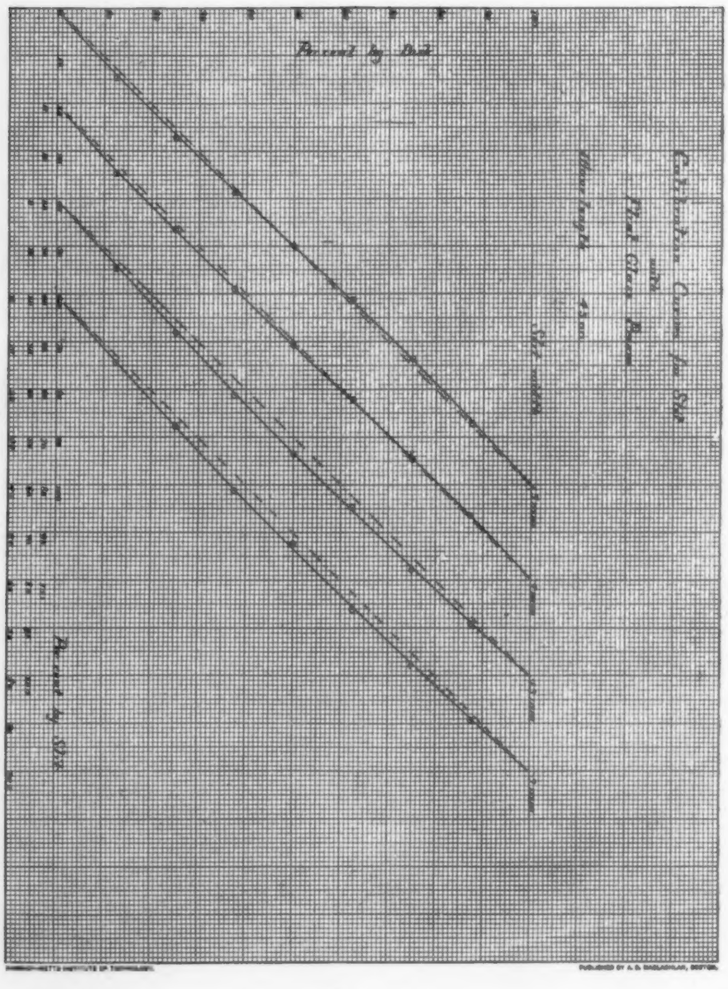

FIG. 11.

both sources were of nearly the same intensity. When one lamp was replaced by a new one no change was produced in the calibration values.

The results may be summed up as follows:

1. The direct ratio law holds for only two points in the spectrum, corresponding approximately to wave-lengths 620 $\mu\mu$ and 570 $\mu\mu$.

2. The variation is greatest in the red and blue parts of the spectrum in the order named, and least and of opposite sign in the yellow.

3. It increases as the slit increases in width,

4. It decreases as the refractive power is increased.

5. It is practically independent of the relation of the two sources when they do not differ by more than 50 per cent.

6. The calibration curves are sufficiently reliable to be used to correct readings when the slit method is adopted for making photometric measurements.

In conclusion the author desires to thank Professors Brace and Moore for valuable suggestions that enabled him to carry out this work.

PHYSICAL LABORATORY,
UNIVERSITY OF NEBRASKA,
December 1899.

ON THE ESCAPE OF GASES FROM PLANETARY ATMOSPHERES ACCORDING TO THE KINETIC THEORY.

By S. R. Cook.

The conditions for the existence of atmospheres on the planets, including the escape or retention of the lighter gases as hydrogen and helium by the Earth, are problems which have been recently discussed in a memoir by Dr. G. Johnstone Stoney.[1]

In these discussions Dr. Stoney has omitted the determination by the Kinetic Theory of the relative number of molecules which would have a velocity sufficient to enable them to escape from the Earth or planet, providing there be no retarding media. This velocity will be termed the critical velocity.

The purpose of the present paper is to apply Maxwell's distribution of velocities to the subject.

In order that any molecule may escape from the atmosphere of any member of the solar system, it must have a normal velocity equal to or greater than the velocity it would acquire in falling from infinity to its present position; and it must be so situated in the atmosphere that it will not be hindered by any impact while going from its present position to infinity.[2]

Since the mutual relations as to temperature, pressure, and density have not been sufficiently accurately determined for the limit of the Earth's atmosphere to allow satisfactory computations to be based on them, I shall compute the number of molecules that, with a critical velocity, will pass through the outer surface of a spherical shell surrounding the Earth and whose thickness is $r\lambda$, where λ is the mean free path of all the molecules in this layer, and $r\lambda$ that of all those escaping. This will be the number of molecules that will leave the atmosphere

[1] This JOURNAL, **7**, 25, January 1898, Royal Dublin Soc., Vol. VI, Part 13.

[2] It must be borne in mind that molecules may be moving in a medium which exerts a retarding influence.

under the assumption that this spherical shell is the outside limit of the atmosphere and that the temperature and density of the shell are constant.

For the Earth the number will be computed under the following conditions:

1. For a spherical shell at the Earth's surface at a mean temperature of 5° C.

2. For a spherical shell 200 km from the Earth's surface at a temperature of –66° C.

3. For a spherical shell 20 km from the Earth's surface at a temperature of –66° C.

4. For a spherical shell 50 km from the Earth's surface at a temperature of –180° C.

The conditions specified in (1) are obviously the most favorable for the escape of the molecules under the assumption that the spherical shell is the outside limit of the atmosphere. The conditions specified in (2) are those assumed in the memoir by Dr. Stoney.[1] The conditions specified in (3) are the results of recent experimental data from balloon ascensions at Paris and Berlin, kindly furnished me by Professor Cleveland Abbe. Those specified in (4) are based on results obtained from theoretical considerations by Ferrel.[2]

The number of molecules having a velocity between c and $c + dc$ is

$$dN_0 = \frac{4N_0}{\bar{c}\sqrt{\pi}} \cdot \frac{c^2}{\bar{c}^2} \cdot \epsilon^{-\frac{c^2}{\bar{c}^2}} dc.$$

Where c is any velocity, $\bar{c}$ is the mean velocity, and N_0 the number of molecules.

The number of molecules having a velocity of $r\bar{c}$ or greater for

$$r \geqq 1 \text{ is } N^1 = N_0 K,$$

where

$$K = 2\epsilon^{-x^2}\left(x + \frac{1}{2x} - \frac{1}{4x^3} + \frac{3}{8x^5} - \frac{15}{16x^7} + \right)$$

and

$$x = \frac{2r}{\sqrt{\pi}}.$$

[1] *Loc. cit.*, p. 1.

[2] FERREL, *Recent Advances*, p. 37.

For all calculation made for the Earth's atmosphere r is greater than 5, so that K becomes a rapidly converging series. The value of the series after the third term being less than 0.1 per cent.

If N is the number of molecules in the limiting spherical shell of thickness $r\lambda$, and if $r\bar{c} = c_1$ is the critical velocity, then NK will be the number of molecules having a velocity equal to or greater than c_1.

If n^1 is the number of molecules in unit volume and R_1 the radius of the spherical shell of thickness $r\lambda$,

$$N = 4\pi R_1^2 r\lambda n^1.$$

Since the critical velocity c_1 is equally probable in all directions, in order to find the number of molecules that will pass through the outer surface of the spherical shell with a velocity c_1 or greater, we determine $\bar{c}_1$, the mean velocity of the molecules having a velocity between c_1 and infinity. Since any molecule, whose component velocity normal to the shell is $c_1 = \bar{c}_1 \cos\theta$, will escape, the proportion of those that will escape during time t_1 will be $\phi : 4\pi$, t being the time of the mean free path $r\lambda$ of these molecules, and ϕ being the solid angle of aperture 2θ. Hence, to the first order of approximation, the number of molecules that will escape in time t_1 will be

$$N_2 = 2\pi R_1^2 r\lambda K n^1 (1 - \cos\theta) \tag{5}$$

The number that will escape in any other time T will be

$$N_3 = 2\pi R_1^2 r\lambda K n^1 (1 - \cos\theta) \cdot \frac{T}{t_1} \tag{6}$$

This formula gives a maximum limit to the number of molecules that will pass outward from the limiting shell with a normal velocity equal to or greater than the critical velocity c_1.

The quantities on the right hand side of equation 6 can all be determined from the Kinetic Theory, except the critical velocity which can be obtained from the formula

$$c_1^2 = 2a\frac{R^2}{R_1}$$

where a is the total acceleration, R is the radius of the Earth, and R_1 is the distance from the center of the Earth to the molecule.

Since N_3 is a linear function of n^1, r_1, $1/t_1$, K, R_1^2 and $(1-\cos\theta)$, the relative variation in N produced by these factors can readily be determined. The value of n^1 as determined by different observers varies from 6×10^{18} to 10×10^{18}, the latter value being used in the calculation. Similarly t_1 and λ will have a corresponding range of variation. The variation of R_1 arising from the different assumed heights of the limiting shell are contained in the tabulated results. Since $\theta=\cos^{-1}\frac{c_1}{\bar{c}_1}$. The variation in $(1-\cos\theta)$ will depend upon the variation in $\frac{c_1}{\bar{c}_1}$, which, from inspection of the curve of distribution of velocities beyond the critical velocity, makes $\frac{c_1}{\bar{c}_1}$, approximately a constant ratio. However, the greatest value consistent with the problem was selected for $\bar{c}_1$, making the value of $(1-\cos\theta)$ a maximum.

Since $c_0=\frac{1}{3}\sqrt{\frac{p}{\rho}}$ and $\bar{c}=\bar{c}_0\sqrt{\frac{T}{273}}$, where $\bar{c}_0$ is zero temperature, any variation in $\bar{c}$ will depend on the variation in the pressure, p, the density, ρ, and the absolute temperature, T, for which $\bar{c}$ is computed. p and ρ are very accurately determined quantities. The absolute temperature T was selected for each assumed spherical shell, such that $\bar{c}$ would be a maximum value,[1] $c_1=2a\frac{R^2}{R_1}$, where a was taken as 981.5 cm, R as 6378 km, and R_1 varied from 6378 to 6578 km, depending on the height of the spherical shell. Since $r=\frac{c_1}{\bar{c}}$, the variation in n_3, produced directly by r will be small; but as r enters into K as an inverse exponent of the second degree, as a factor, and in the term used to the fifth degree, any slight variation in r will produce a very great variation in the value of K, and as the value of r has been

[1] The following table gives some of the results of recent balloon ascensions:

November 14, 1896,	height,	15,000 meters;	temperature,	—60° C.	Paris.
December 4, 1894,	"	10,000 "	"	—52° C.	Berlin.
October 20, 1895,	"	15,000 "	"	—70° C.	Paris.
March 29, 1896,	"	14,000 "	"	—63° C.	Paris.

determined in each case by the selection of the value of the absolute temperature and the distance of the spherical shell from the Earth, the value of K will depend upon their selected values. I have at all times selected such values of these factors as were consistent with a special problem and which would make N_3 a maximum.

In the following table, column one gives the conditions specified on page 2; the second column the critical velocity in kilometers per second; the third column the ratio of the critical velocity to the mean velocity; the fourth column the number of molecules that will escape during the time of the mean free path; the fifth column the number of cubic centimeters of the gas that will escape in one year; and the sixth column gives the number of cubic centimeters that will escape in 10^7 years.

The results which Professor Bigelow obtained by working over the data of all balloon ascensions gives in general a fall of 65° to 70° for 16,000 meters. I have taken the temperature —66° C. at a height of 20 km, and the value 1841 m sec. for $\bar{c}_0$ for hydrogen.

HYDROGEN.

Condition	c_1	r	N_3 $T=t_1$	N_3 in cc $T=1$ year	N_3 in cc $T=(10)^7$ years
1	11.	5.92	$80.24\,(10)^{19}$	$33.04\,(10)^{8}$	$33.04\,(10)^{15}$
2	10.5	6.55	$45.15\,(10)^{4}$	$23.58\,(10)^{-7}$	23.58
3	10.98	6.85	$72.61\,(10)^{9}$	$54.28\,(10)^{-2}$	$54.28\,(10)^{5}$
4	10.90	10.14	$66.8\,(10)^{-4}$	$43.5\,(10)^{-15}$	$43.5\,(10)^{-8}$

HELIUM.

1	11	8.37	19.8	$10.34\,(10)^{-11}$	$10.34\,(10)^{-4}$
2	10.5	9.27	$44.5\,(10)^{-18}$	$22.10\,(10)^{-31}$	$22.10\,(10)^{-24}$
3	10.98	9.78	$50.15\,(10)^{-19}$	$26.73\,(10)^{-30}$	$26.73\,(10)^{-23}$
4	10.90	14.5	$19.27\,(10)^{-88}$	$91.6\,(10)^{-93}$	$91.6\,(10)^{-86}$

The results here given were computed from equation (6). The values for N_3 are computed for a hydrogen and a helium atmosphere, and they represent the amount that would escape

from an atmosphere of hydrogen or helium under the specified conditions and bounded by the respective spherical shells.

Condition (4) represents most nearly the conditions of the outer limit of the atmosphere; $43.5\ (10)^{-8}$ cc of hydrogen would escape from a hydrogen atmosphere thus bounded and conditioned during 10^7 years, and $91.6\ (10)^{-8}$ cc of helium would escape from a helium atmosphere in the same time similarly conditioned. Since under these conditions a very small fraction of 1 cc of hydrogen would escape from a hydrogen atmosphere in 10^7 years, it is evident that the amount which is actually escaping from the Earth's atmosphere, if any, is insignificant. The amount of helium escaping would be zero.

There are approximately 10^{24} cc of air in the atmosphere, and under the most favorable condition less than 10^{10} cc of hydrogen would escape from an atmosphere of hydrogen whose outer layer was 5° C. and whose density was the density of hydrogen at atmospheric pressure, in one year. It would under these conditions take 10^{14} years for an amount of hydrogen equal to the Earth's atmosphere to escape. If we reduce the value of r from 5.92 to 5, we find approximately that 10^{15} cc of hydrogen would escape per year, and it would take 10^9 years for an amount equal to the atmosphere to escape under those conditions.

I shall now apply the results obtained for an atmosphere of hydrogen on the Earth to the atmospheres of the Moon and planets. I shall use the values for the minimum velocity of escape given by Dr. Stoney, as they will be sufficiently accurate for a very close approximation.

The relation between the molecular velocities and the absolute temperature is

$$\frac{c_1^2}{c_0^2} = \frac{T_1}{T_0}.$$

The following table gives the value of the temperature of the outer layer of the atmosphere which would enable an atmosphere to escape at the same rate as hydrogen would escape from an atmosphere of hydrogen whose outer layer is conditioned as in

(1) and whose critical velocity is $5\bar{c}_1$, the number of molecules in the atmosphere being the same as in the Earth's atmosphere, and the time allowed for the atmosphere to escape being 10^9 years.

TABLE II.

			Hydrogen		Air		Carbondioxide	
	c_1	$\bar{c}_t$	t	r_1	t	r_1	t	r_1
Moon......	2.380	.476	—256°	1.24	—10°	4.7	274°	6.6
Mercury....	4.468	.8936	—209°	2.4	894	9.2	1371	12.4
Venus......	9.546	1.909	20°.5	5.185	5031	19.27	7403	26.5
Mars........	4.803	.960	—195°	2.66	1139	9.9	1807	13.3
Earth......	10.5	2.100	291°	5.7	9937	21.7	14447	29.17

c_1 is the critical velocity in kilometers per second; $\bar{c}_t$ is the mean velocity of the molecule in km sec. $=c_1/5$; t is the temperature of the outer layer of the atmosphere in centigrade degrees, and r is the ratio of the critical velocity to the mean velocity.

This table shows that on the Moon an atmosphere of hydrogen would escape with its outer layer at $-256°$ C.; an atmosphere of air at $-10°$ C., and an atmosphere of carbon dioxide at 274° C. . For Mercury the outer layer would necessarily be $-209°$ C., 894° C., and 1371° C. for hydrogen, air, and carbon dioxide respectively. For Venus the outer limit of the respective atmosphere would be 20.5° C., 5051° C., and 7403° C. The temperature of the outer limit of the Earth's atmosphere, in order that the respective atmospheres would escape, would be, for hydrogen 291° C., for air 9937° C., and for carbon dioxide 14447° C. For Mars the outer limit of the respective atmospheres would be $-195°$ C., 1139° C., and 1807° C.

In order to draw any more definite conclusions in regard to the atmosphere of the Moon and planets, it will be necessary to know the temperature of the planets and the temperature gradient of their atmosphere. It appears, however, from the above figures, that an atmosphere like that of the Earth would not remain on the Moon, but would remain on any of the planets

mentioned. The Earth and the major planets would not only retain an atmosphere of nitrogen and oxygen, but also hydrogen and helium.

My thanks are due to Professor D. B. Brace for many valuable suggestions, and also to Professor Cleveland Abbe for valuable data and references.

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ON A BALANCING RHEOSTAT FOR BOLOMETERS.

By P. G. NUTTING.

PROFESSOR F. L. O. WADSWORTH some time ago described in this JOURNAL[1] a device for making fine bridge adjustments by means of a double contact arrangement for one of the battery terminals. With the best contacts we could construct, however, we found the error due to varying contact too great to make the arrangement available for use with our differential bolometer in the determination of absorptive indices. We finally hit upon the device of connecting *both* galvanometer terminals by variable contact to the bridge angles instead of only one as is usually done. The bridge junction at one galvanometer terminal is made of rather fine wire, of just sufficient resistance to overcome the inequalities of the bolometer arms. The other bridge junction at the remaining galvanometer terminal is made of large wire of very low resistance, and serves for the delicate final setting. The resistance of this whole wire may be made equal within about 1 mm, or less, to that at the opposite terminal, but for rapid work it is conveniently of a resistance sufficient to balance the greatest bolometric variations expected. It is further convenient to choose the balancing arms of such diameter that their lengths may be made equal, and not more than 20 cm. These are stretched side by side on an ebonite plate, and a millimeter scale inlaid under the one where the final adjustment is made. The galvanometer terminal makes contact with this wire by means of a sharpened piece of No. 8 wire, which is frequently repointed. The rough adjustment is made first, and that terminal fastened with wax. If a null method is not used, it is packed and enclosed to guard against temperature waves, leaving only the large, low resistance wire exposed for adjustments.

For the most delicate bolometric work there remain only

[1] This JOURNAL, 5, 268–276, April 1897.

thermal currents and variations in resistance with room temperature to be eliminated in the balancing rheostat. Working with a null method, we make all external connections of copper wire, rheostat included. Junctions are made close fitting and soldered. The only thermal junctions are at the ends of the platinum receiving strips, and these are enclosed within the bolometers themselves. Changes in resistance with room temperature are of no consequence when a null method is used. For other work, nickeline or manganin is used for one rheostat wire, or else a temperature correction applied. This may easily be done to within the probable error due to other causes.

The balancing rheostat used here in connection with the double bolometer is constructed of copper wires of about 0.005 and 0.0003 ohms, each 10 cm long. The adjustment of the galvanometer terminal can be made to within $\frac{1}{10}$ mm; readings of its position are unnecessary using the null method. $\frac{1}{10}$ mm on the larger wire corresponds to 0.0000003 ohm. This is but 0.000000015 of the resistance of the bolometer arms. Calling the temperature coefficient of platinum 0.005 per degree, a temperature change of 0.000003° in the bolometer strips can be detected. For these tests a commercial d'Arsonval galvanometer and an old 1.5 volt, 20 ohm Leclanche cell were used. Our new, specially constructed, light suspension, Paschen galvanometer is sensitive to a current of less than 1×10^{-11} amperes. Using only $\frac{1}{8}$ the former current, and a larger and longer rheostat wire, the sensitiveness is increased about one hundred times.

Excellent quantitative work may be done with the same arrangement. A simple computation will show that in the determination of a small percentage variation in resistance, the coefficient is certain to as many significant figures as is the resistance itself. For example, when copper wires of 2.002 and 2.006 ohms resistance were surrounded with gasoline, a shift of 22.4 mm in the rheostat wire was necessary to restore equilibrium. This shift corresponded to 0.00000683 ohm, or 0.000167 of 1 per cent. change in resistance for each wire. Changes due to temperature were eliminated by stretching the

four very nearly equal wires forming the bridge arms side by side in a heavy brass tube, and only exposing opposite arms to the dielectric.

By a somewhat similar method, the temperature coefficient of some new resistance wire sent us from Germany was found to be — 0.00000271 per degree.

It is hoped that the above may be of use to a few of the many investigators now using bolometers in research.

UNIVERSITY OF CALIFORNIA,
November 1899.

SOLAR ECLIPSE PROBLEMS.

By George E. Hale.

The writer, as secretary of the Eclipse Committee of the Astronomical and Astrophysical Society of America, has received so many requests for suggestions from intending observers of the coming total eclipse that it has seemed advisable to bring together, for the convenience of those who may not have ready access to astrophysical literature, various data bearing upon some of the more important problems which this eclipse may help to solve. As no total eclipse of the Sun has been visible in the United States since 1889 it may be expected that many individuals and institutions will be anxious to take advantage of the present opportunity. Considered merely as a spectacle, the eclipse will amply repay those who may journey far to see it. But if the maximum scientific return is to be derived from this rare phenomenon, every intending observer must familiarize himself with the results obtained at previous eclipses, devote much time to the consideration of the questions still demanding solution, and give particular attention to the design of the apparatus required for the work selected. It fortunately happens that valuable scientific results can be obtained with simple and inexpensive instruments. The unsolved problems, too, are numerous and varied, offering a wide field for choice, and appealing to the individual tastes of many observers. It is only necessary to chose intelligently, basing the decision upon one's previous experience in observational work and a consideration of available instruments. If possible, there should also be some measure of coöperation between different parties, and here the Eclipse Committee stands ready to lend its aid.

The eclipse will occur on May 28, 1900, between 8 A. M. and 9 A. M. The central line of the shadow will cross the states of Virginia, North Carolina, South Carolina, Georgia, Alabama,

Mississippi, and Louisiana, the duration of totality decreasing from $1^m 46^s$ on the Atlantic seaboard to $1^m 18^s$ near New Orleans. The advantage due to a longer duration of the total phase is more than offset, however, by the much greater probability of cloudiness at points near the ocean, as shown by the survey of cloud conditions recently made under the supervision of the United States Weather Bureau. From these observations it appears that the chances of fair weather are best on the highland of the southern portions of the Appalachian Mountains.[1] It is to be hoped, however, that all of the stations will not be confined to this region, as there are many reasons why they should be distributed all along the line of totality.

In preparing a plan of operations it is first necessary to remember that the eclipse will be a short one, even for those who may view it from the Atlantic coast. Other things being equal, it therefore will not be desirable to undertake observations which cannot be successfully completed within a space of little more than a minute. Thus it would hardly be advisable to attempt to photograph the fainter portions of the spectrum of the corona on this occasion, and it is exceedingly doubtful whether it would be worth while to repeat the hitherto unsuccessful attempts to measure spectroscopically the period of rotation of the corona. Although some reference to such experiments may be found in this paper, it will probably be best to reserve these observations for the Sumatra eclipse, when totality will last over six minutes. On the other hand, the spectrum of the reversing layer can be photographed next May to good advantage, especially, as Mr. Evershed has pointed out, at stations not far removed from the edges of the shadow path. Many other phenomena can be advantageously studied during the total phase, provided the observer will limit himself to some well-defined piece of work, which must not be too comprehensive in its scope.

[1] See *Weather Bureau Bulletin*, No. 27. In addition to statistics of cloudiness, this bulletin contains much valuable information regarding suitable sites for stations, hotel accommodations, etc.

The suggestions offered in the following paragraphs make no pretensions to fullness of scope or adequacy of treatment. They are in no sense intended to take the place of "Instructions" to eclipse observers, and it is not supposed that they will be of service to experienced astrophysicists, except possibly in recalling certain matters which are sometimes overlooked. To amateurs possessing small instruments, and to others who have had little practical experience in solar work, it is hoped that they may be of some value.

1. NAKED-EYE DRAWINGS OF THE CORONA.

The experience of previous eclipses has shown that drawings of the corona for the most part serve no useful purpose, unless it be to illustrate the personal peculiarities of the draftsman. At the present time, when everyone who possesses a suitable camera may secure a photograph which can be relied upon, it seems quite superfluous to devote the brief time of totality to sketching. Naked-eye observations are specially referred to here. If drawings are made it should be only after long and faithful drill, and the more limited the region drawn the better.

2. DRAWINGS AT THE TELESCOPE.

What has been said regarding naked-eye drawings applies with much less force to drawings made with the aid of a telescope. According to the testimony of the best observers the most successful large-scale photographs hitherto made fail to reveal the most interesting details of the coronal structure. These details can be seen with telescopes of from three to six inches aperture, and an observer practiced in rapid sketching should be able to secure a valuable record if he confines his attention to a limited region. Concentration of this kind, especially if the field of view is small, should eliminate many of the distractions which so seriously interfere with naked-eye observations. It is particularly to be desired that some observers should devote their attention to the lower corona in the immediate

vicinity of large prominences. Those who would be likely to distinguish an active prominence by a glance at its form would do well to record the coronal details near it. Others should sketch the polar streamers close to the Moon's limb, and attention may also be given to the structure of the outer corona, both in the polar and equatorial streamers. It is evident that coöperation of some sort is desirable in this work, in order that the various parts of the corona may receive proper attention.

3. COLOR OF THE CORONA AND PROMINENCES.

Reliable information regarding the color of the prominences and various parts of the corona is much to be desired. But if the record of observations is to be of real service, it must consist of something more definite than fanciful similes, such as are likely to be called forth by the excitement of the moment. The colors must be well and clearly defined, and if possible referred to some standards of color, prepared for the purpose. If a telescope is not employed the observations will be much facilitated by the use of a field or opera-glass, provided instrumental color is not introduced in this way. The presence of the prominences greatly interferes with determinations of the color of the corona, and in such observations it would be advantageous to occult them with a small disk in the focal plane of the telescope.

The importance of noting the color of the prominences rests upon the fact that Tacchini and others have observed the so-called "white prominences," of which an adequate explanation has yet to be offered. The subject has been discussed by the present writer in an article entitled "The Effect of a Total Eclipse of the Sun on the Visibility of the Solar Prominences,"[1] where Tacchini's observations are fully described. To mention only a single case, it may be said that at the eclipse of 1886 Tacchini distinctly saw an enormous white prominence, which appeared on the photographs of the inner corona, but could not be seen with the spectroscope in the $H\alpha$ line either before or

[1] This JOURNAL, 3, 374, 1896.

after totality. In the article referred to I have suggested that the white color of such a prominence may be due to the comparative faintness of the less refrangible lines in its spectrum, whereas the calcium lines H and K were of their normal intensity. Objective prism photographs of the chromosphere and prominences have indeed shown marked variations in the relative intensity of prominence lines, but it is not yet certain that a "white prominence" could be caused in this way. From this point of view it would be well to compare together the colors of a small number of prominences, as seen in a field-glass or telescope, noting the degree of redness in each case. Such observations should of course be supplemented by work with the spectroscope whenever possible. Photographs of the "flash" spectrum made with an objective prism will be very useful in this connection. If circumstances permit, the chromosphere and prominences will be photographed in the K line with the large spectroheliograph of the Yerkes Observatory at the time of totality for purposes of comparison.

4. SMALL SCALE PHOTOGRAPHS OF THE CORONA.

Every possessor of a suitable camera is equipped with apparatus sufficient to give valuable results in photographing the corona. As Mr. Maunder has pointed out in his interesting discussion of the results obtained at the recent Indian eclipse, with lenses of focal length not greater than five feet and ratio of aperture to focal length not less than $\frac{1}{15}$ it is not necessary to have an equatorial mounting for the camera when the exposure does not exceed one half second. It must not be supposed, however, that it is only necessary to point any camera at the Sun and make a snap exposure. Such a procedure might possibly produce a good picture of the corona, but the most valuable results will be obtained by those who observe a few simple precautions:

a. It is advantageous to use lenses of considerable focal length (from 30 to 60 inches) provided the angular aperture is not too small. Such lenses should show the corona on a scale

large enough to bring out many important details. Very short focus lenses will be useful for long-exposure photographs of the coronal streamers, if mounted equatorially and moved by clock-work.

b. Light and unsteady camera boxes should not be employed. A strong wooden box of the proper length is much more suitable than a camera having leather bellows.

c. It is of great importance that the camera box should be very solidly mounted, so that it will not be affected by the wind and not easily set into vibration by any cause. Unless an equatorial mounting is to be used it will be advantageous to screw the wooden camera box to the top of a post firmly planted in the ground. The optical axis of the lens should point toward the Sun at the middle of totality. Unless the focal length of the lens exceeds 5 feet, or the diameter of the field is very small, it will be unnecessary to change the position of the camera during totality.

d. The lens can be most accurately focused by photographing star trails, a series of exposures being made for different positions of the lens. After the best focus has been ascertained the lens should be firmly clamped in place.

e. The results obtained at the Indian eclipse demonstrate the great advantage of using multiple coated plates. It will be best to employ triple or quadruple coated plates, and as a further safeguard the plates may also be given a suitable backing in order to still further reduce the danger of solarization. With such plates it is possible with an equatorially mounted camera to obtain the faint outer extensions of the corona without serious interference from the bright inner corona. However, too much should not be left to the plates, for the care exercised in development is of the greatest importance.

f. With the most rapid plates it would appear from the results discussed by Mr. Maunder that for an aperture of $\frac{f}{15}$,[1]

[1] For other angular apertures the exposure may be considered to vary inversely as $\left(\frac{f}{a}\right)^2$.

exposures of from $\frac{1}{60}$ second to $\frac{1}{45}$ second will suffice for the prominences and $\frac{1}{10}$ second to $\frac{1}{5}$ second for the inner corona. Such short exposures can of course be given only with the aid of a suitable shutter. A greater extent of the corona may be obtained with exposures from $\frac{1}{2}$ to $\frac{3}{4}$ second. Mr. Maunder believes that in order to obtain a photograph of the corona as a whole the exposures should not exceed one second. It is evident that most of the exposures so far referred to are easily within the reach of those who employ fixed cameras. If it is desired to secure the great extensions of the corona, which at the Indian eclipse were traced to a distance of nearly fourteen lunar radii from the Sun, the camera must be mounted on an equatorial telescope provided with a driving-clock. In this case the exposure might well continue throughout the entire time of totality, but great care should be taken to cap the lens several seconds before the expected reappearance of sunlight. It is desirable to have several lenses of different focal lengths carried by the same equatorial mounting. The long focus lenses, giving large solar images, should be used in photographing the prominences and the structure of the inner corona, while the short focus lenses may be employed to record the long streamers.

g. Care must be taken to provide for the accurate orientation of all photographs, as it is highly important that the north and south points of the image shall be known. If the plate stands in a vertical position, as it may when a heliostat is used, a fine plumb line hanging immediately in front of it will give satisfactory results. In the more common case, where a camera pointing at the Sun is employed, a series of rapid exposures made with reduced aperture during the partial phase, while the Sun moves across the stationary plate, will serve the purpose.

It is to be hoped that during the coming eclipse cameras in the hands of amateur observers will be distributed all along the line of totality in the United States, in Spain, and in Algeria.

A comparison of such a series of photographs might lead to very valuable results.[1]

5. PHOTOGRAPHS OF THE CORONA DURING THE PARTIAL PHASE.

During the recent Indian eclipse Mrs. Walter Maunder, using a triple coated plate and a Dallmeyer stigmatic lens of 1½ inches aperture and 9 inches focal length, succeeded in securing a photograph of the inner corona 39 seconds after the end of totality. As Mr. and Mrs. Maunder point out in the report to which such frequent reference has been made, it will be advisable to repeat and extend these experiments at the coming eclipse. The problem of photographing the corona in full sunlight is one which has received a great deal of attention, but hitherto without tangible result. It may be doubted whether the use of multiple coated plates will be sufficient of itself to overcome the great obstacle of atmospheric glare, but it is not impossible that the corona can be photographed in this way during partial eclipses. A series of exposures made before and after totality on multiple coated and heavily backed plates should be of value, especially if the image of the photosphere is prevented from falling on the sensitive film. An occulting disk before the plate should therefore be employed if possible. Mr. Maunder believes that with an aperture of $\frac{f}{15}$ the exposures out of totality should range from $\frac{1}{5}$ second down to $\frac{1}{50}$ second.

6. PHOTOGRAPHIC SEARCH FOR POSSIBLE INTRA-MERCURIAL PLANETS.

The possibility that planets may exist within the orbit of Mercury has led to many attempts to detect such objects under the favorable conditions presented by a total solar eclipse. As the unsuccessful visual observations made at previous eclipses tend to indicate that no very bright planet is likely to be found, preparations should be made to record all objects, down to the

[1] In this connection one cannot do better than consult the *Report on the Indian Eclipse of 1898*, recently issued by the British Astronomical Association. Many of the above suggestions are derived from this source.

lowest attainable magnitude, in the region about the Sun. Except by a most fortunate chance nothing can be expected to result from visual observations. Even if the region were divided among a great number of observers, little could be done in the brief space of totality, and disputes would be likely to arise on account of the uncertainties inseparable from hurried work. It would therefore seem inadvisable to waste effort on visual observations, but a thorough photographic search should undoubtedly be made. In undertaking such a search the following considerations are among those which should be borne in mind:

a. The lens should combine the greatest possible rapidity of action with the largest possible field. Good portrait lenses having an aperture of about $\frac{f}{5}$ should give well-defined star images over a field about nine degrees in diameter on a flat plate. Experiments recently made by Professor Wadsworth and Mr. Brashear show that the diameter of the field can be greatly increased by the use of concave plates of suitable curvature. It is advisable to employ lenses in pairs, in order that suspected images may be verified on an exactly similar plate made with another lens. In addition to portrait lenses it will probably be advantageous to use rapid rectilinear lenses giving very large fields.

b. The suitability of a lens, the best exposure time, and the proper development can all be ascertained by photographing stars under conditions as nearly as possible like those prevailing during totality, when the general illumination will probably be not very different from that of a clear sky half an hour after sunset. The maximum exposure given should be a few seconds less than the computed duration of totality, for in order to avoid the possibility of complete loss it will be advisable at the eclipse to cap the lens an appreciable time before the Sun is expected to reappear. Such experiments will enable one to select the lens best adapted for the purpose, to determine approximately the exposure time giving the greatest number of stars, and to

form an idea of the number of stars likely to be recorded during the eclipse. Professor W. H. Pickering, who has great confidence in the photographic method, finds from experiment that it should be possible, under eclipse conditions, to photograph stars as faint as the sixth magnitude.

c. The camera should be carried by a good equatorial mounting, provided with driving-clock. Care should be taken in adjusting the instrument, for without perfect following the maximum photographic effect of course cannot be obtained.

d. Those who possess large portrait lenses of great angular aperture, but lack a suitable mounting, will do well to try photographing star trails under the conditions named above. It is probable that even with a fixed camera the chances of success would be decidedly greater than in the case of visual observations.

e. It goes without saying that the most rapid plates obtainable should be used. How far it will be safe to push the development can be roughly determined from the preliminary experiments.

f. In order to cover the largest possible area of sky it is very desirable that parties planning to undertake this work should coöperate, each devoting its attention to a particular region. All persons provided with suitable instruments, who are willing to participate in a general plan of operations, are invited to address the Eclipse Committee.

It may be added that Professor Newcomb, whose suggestions are embodied in the above paragraphs, considers a photographic search for possible intra-Mercurial planets as one of the most important pieces of work that can now be attempted during total eclipses of the Sun.

7. LARGE SCALE PHOTOGRAPHS OF THE CORONA AND PROMINENCES.

Portrait lens photographs of the corona, while of great value for certain purposes, are on too small a scale to permit the finer structure of the corona and prominences to be recorded. For this purpose the largest possible images are required. The

results obtained at recent eclipses with objectives of great focal length indicate the value of this method. The lens may be pointed directly at the Sun and used in a fixed position with a moving photographic plate, or it may be mounted with its axis horizontal, and supplied with light by a heliostat or coelostat. A double mirror heliostat or a coelostat should be used in preference to a single mirror heliostat (unless this be of the polar form), as no large instrument of the latter type, so far as the writer is informed, has ever been made to drive accurately. If the mirrors show any tendency to distortion by the Sun's heat they should be screened until a few seconds before totality. Of the lens it may be said that it is doubtful whether any advantage will result from the use of focal lengths much greater than 75 or 100 times the aperture. A consideration of the size of the silver grains would indicate that with perfect seeing the full visual resolving power of the objective should be realized when the focal length is about 50 times the aperture. Excellent results have, however, been obtained with objectives whose focal lengths are as much as 120 times the aperture. But it must not be forgotten that for an objective of given aperture the exposure time and the effects due to poor seeing and inaccuracies of the coelostat or heliostat increase rapidly with the focal length. Hence one should not go too far in his desire to secure a large focal image of the corona.

It is probable that a photographic objective of comparatively short focus (say $\frac{f}{16}$), if used in conjunction with a suitable enlarging lens, would give results but little inferior to those obtained with the apparatus described above. This would have the advantage of requiring only a small equatorial mounting, but it might be difficult to avoid setting the instrument into vibration when changing plate-holders. All apparatus used in large scale photography must of course have stable foundations and adequate protection from the wind.

It will probably be advantageous to use multiple coated and backed plates, particularly for the longer exposures. The

approximate lengths of exposures may be calculated from the data given in section 4.

The device employed at the Indian eclipse by Mr. Burckhalter, for the purpose of giving to each part of the corona an exposure inversely proportional to its brightness, merits more general use. It consists of a revolving disk, mounted concentrically with the Sun's image, and admitting light to the plate through an aperture of suitable form. The width of the opening at a given point cannot be exactly known until an expression giving the (photographic) brightness of the corona as a function of the distance from the Sun's limb has been found. Nevertheless, data sufficient for practical purposes are available, and there should be no difficulty in repeating Mr. Burckhalter's experiments.

8. DISTRIBUTION OF "CORONIUM."

It is a matter of considerable importance to determine whether "coronium"—the gas whose spectrum is characterized by the bright green line formerly known as "1474 *K*"—forms a structureless envelope about the Sun, or conforms in appearance to the corona. In other words, does the bright green line pass without marked change of intensity over coronal rifts and streamers, as Tennant long ago found it to do? It is evident that this question may be tested in several ways. One of the simplest is that employed by Maunder at the Indian eclipse of 1898. A small direct-vision prism was inserted in one of the eyepieces of a binocular of about two inches aperture. It was thus possible to compare the corona, as seen directly with one eye, with its monochromatic green image, as seen with the other. Thus the distribution of coronium could be ascertained with rapidity and certainty. Unless adequate shade glasses are used the Sun should not be observed with such an instrument before the beginning of totality, as the eye is likely to be temporarily injured by the brilliancy of the image. This was Mr. Maunder's experience, and as a result he was able to see the faint green image only after mid-totality had passed. No trace of rifts or rays could be distinguished by this observer. On account of its

small size and convenience, a prismatic binocular will be very suitable for persons who wish to make useful observations, but are not in a position to provide elaborate and expensive instruments. Care must be taken, however, to secure an efficient binocular and direct-vision prism. The magnifying power of the binocular should be sufficient to show the coronal structure clearly and distinctly. The instrument, with prism in place, can be tested by the aid of an alcohol or Bunsen flame, colored with lithium or strontium chloride, and partially screened by a piece of cardboard in which openings are cut having the same angular magnitude as the coronal rays.

It would be interesting to examine with such an instrument the green coronal image in the immediate vicinity of a prominence. An objective prism or grating, or a small direct-vision prism, used in conjunction with a telescope of three or four inches aperture, would be more suitable for this purpose, however, on account of the larger scale of the image.

9. POSITION OF THE GREEN CORONAL LINE.

It has been recently shown by Lockyer, Campbell, and others that the green coronal line photographed at the Indian eclipse of 1898 did not correspond in position, as it had previously been supposed to do, with the line falling at 1474 on Kirchhoff's map of the solar spectrum, Undoubtedly the best way to accurately determine the wave-length of the green coronal line at the coming eclipse is by photography, but direct visual comparisons of the chromospheric and coronal lines will also be of value. Any spectroscope that easily separates the D lines will serve for the purpose, but a dispersion of two or three prisms would be advantageous. The spectroscope should be attached to a telescope of from four to six inches aperture, provided with good slow motions and driving-clock. The slit should be made radial at the computed point of second contact, and set in the focus of the telescope corresponding to λ 5317. The micrometer of the spectroscope should be provided with a rather coarse wire, extending only half way across the field, and fined down to a

sharp point. At the beginning of totality this wire may be set so as to occult the bright chromospheric line at λ 5317 (1474 *K*). A few seconds later the green coronal line should become visible, and it may be seen at once whether it coincides in position with the wire. If time permits, micrometer settings may be made to determine the wave-length of the coronal line. Or the same instrument may be employed in an attempt to trace the green line across the coronal rifts.

10. PHOTOGRAPHS OF THE SPECTRUM OF THE CHROMOSPHERE.

The importance of securing a complete record of the spectrum of the base of the chromosphere is so great that no pains should be spared to surpass the admirable results recently obtained in India. So far as the writer knows, the numerous lines of the green and yellow flutings of carbon vapor, which can be seen at the Sun's limb on any good day with the spectroscope attached to the 40-inch Yerkes telescope, have not yet been observed or photographed during eclipses. This indicates that the spectrum of the "flash" is probably far more complicated than even the best photographs show it to be, and thus emphasizes the fact that too much attention cannot be devoted to this field of eclipse research. The brief duration of the flash is the principal obstacle to success. For this reason Mr. Evershed's suggestion that stations lying only a few miles within the north and south boundaries of the shadow path be selected for this work is an excellent one. At such points the duration of totality may be only one third that on the central line, but during this entire time the spectrum of the chromosphere would be visible. Thus photographs taken at second and third contacts would give the combined spectra of the lower and upper strata, while exposures made at mid-eclipse would record only the higher layers. Observers who wish to give special attention to the smaller prominences and the lower corona could also work to advantage at such stations, but all other observations should be made on or near the central line.

There can be little doubt that of all forms of spectroscopes suitable for photographing the spectrum of the flash the objective prism is the one most likely to give good results. As compared with a slit spectrograph it has certain disadvantages, notably on account of the absence of a slit, which renders the use of a comparison spectrum practically impossible. This defect is rendered less serious by the fact that a sufficient number of well-known lines to serve as standards can be found in the chromospheric spectrum itself. Thus the wave-lengths of the unknown lines can be determined with a considerable degree of accuracy, especially if Hartmann's valuable interpolation formula is used, and the reductions made by the method of least squares. Again, the necessary exposure times are less than with a slit spectrograph of equal resolving power, and the danger of complete failure is very small.

A slit spectrograph, if it is to be comparable in working efficiency with an objective prism, should be of large aperture, while the camera objective should be a triple lens or some other combination capable of giving good definition over a fairly large field. If a tangential slit is to be used the solar image should be as large as possible, and special care should be taken to provide reliable slow motions for use in placing the slit tangent to the Sun's limb. A train of prisms is undoubtedly superior to a grating in an instrument of this kind, unless one is fortunate enough to possess a grating which concentrates most of the light in a single spectrum. The time of exposure may be reduced to a minimum by using a long collimator (of correspondingly large aperture) and a comparatively short camera. The focal length of the camera objective will of course depend on the angular dispersion of the prism train and the linear dispersion desired on the photograph. In general, when it is necessary to reduce the time of exposure to a minimum, it is not advisable to attempt to realize the full theoretical resolving power of the prism train. This could be done only by the use of a very long camera, which would require a great increase in the exposure time.

Summing up, it may be said that the objective prism has a decided advantage in requiring less care in manipulation, and in

showing a long arc of the chromosphere with a comparatively short exposure. It is therefore likely to be used in the majority of cases. Slit spectrographs should be employed only by experienced observers, who must take pains to secure the highest possible degree of efficiency in the design. The chances of photographing the spectrum of an active region in the chromosphere will be much increased if a slit having the curvature of the image of the Moon's limb is substituted for a straight slit. This presupposes that the slit is to be used tangentially, the position which demands the greatest skill on the part of the observer, but the one which may ultimately give the best spectrum of the lowest stratum of the chromosphere. Although a large solar image is needed, brightness should not be sacrificed for size through the use of an objective of very small angular aperture.

Reflecting jaws would facilitate setting the image on the slit. If a radial slit is used much less care is necessary in adjusting the position of the solar image, which may also be smaller in diameter. In this case the plan used by Campbell at the Indian eclipse, of moving the photographic plate by clockwork during the exposure parallel to the spectral lines, should prove effective in giving the spectrum of the various levels of the chromosphere on a single plate. The same method may be adapted to the requirements of a tangential slit or to those of an objective prism by limiting the length of the spectral lines by means of a slit parallel to the length of the spectrum, placed immediately in front of the photographic plate.

A third method of using the spectrograph is to set the slit so that it makes a small angle with the tangent to the Moon's limb at the point of contact. With a fairly large image of the Sun the spectrum obtained in this way on a fixed plate is quite wide enough to differentiate the spectra of the various strata. In observations of the faintest lines in the chromospheric spectrum with the 40-inch telescope this manner of placing the slit is found to give excellent results.

It is to be hoped that someone will use a quartz spectrograph for work in the ultra-violet. For this and other purposes the

valuable combination of polar heliostat and spectrograph used by Newall in India should prove useful.[1]

Little need be said regarding the design of the objective prism spectroscope, unless it be to emphasize the importance of employing a lens corrected for the wave-lengths comprised by the photograph and the advantage of using several prisms with a short focus objective rather than a single prism with an objective of long focus. The dark line spectrum of the cusps should be photographed for use in connection with the reductions.

A direct concave grating spectroscope, similar to that used in 1899 by Mitchell at the Yerkes Observatory and described by him in the June number of this JOURNAL, should give good results in eclipse work. Newall describes in the paper referred to above an objective grating with which he successfully observed the distribution of "coronium" about the Sun.

In all cases it will be advantageous to employ multiple coated and backed plates, and to determine the proper moment to expose for the spectrum of the flash by the aid of a solar spectroscope used visually. A suitable photographic shutter should be provided, and a device for rapidly bringing fresh plates into position would prove of great service.

II. PHOTOGRAPHS OF THE SPECTRUM OF THE CORONA.

As already remarked, the eclipse will be of so short duration that it will be inadvisable to attempt any exhaustive study of the spectrum of the corona. Nevertheless, in view of the importance of definitely determining its wave-length, the green coronal line should be photographed with the highest dispersion it is safe to employ—probably three prisms, with a long collimator and a comparatively short camera. If possible, the chromospheric line at λ 5317 and the solar spectrum should be photographed on the same plate for purposes of comparison. From such a plate an accurate determination of the wave-length of the green coronal line may easily be obtained.

[1] See *Proc. Roy. Soc.*, **64**, 55, 1898.

Further attempts to measure the rotation of the corona will probably be put off until the Sumatra eclipse, but if any work is done it would appear from Campbell's results that the green line should be chosen in preference to all others. In any event the H and K lines should not be used, as the objective prism has shown that they do not belong to the spectrum of the corona.

12. HEAT RADIATION OF THE CORONA.

Direct measurements of the heat radiation of the corona, although attempted at two previous eclipses, have hitherto yielded no satisfactory results. As Professor Very has pointed out in the October 1899 number of this JOURNAL, the question is one of great intrinsic importance, and it has an additional interest because of its bearing on the possibility of mapping the corona without an eclipse. If the corona radiates sufficient heat to produce easily measurable deflections with a bolometer, radiometer, or other sensitive heat-measuring instrument, and if the intensity of heat radiation from the rifts and streamers is sensibly different, there is a chance that it may ultimately become feasible to map the outline of the corona in full sunlight. Discussion of the method would be out of place here;[1] suffice it to say that the principle on which it is based has been proved by laboratory experiments to be sound. Two sets of measures are particularly desirable just at present:

a. Measures of the heat radiation of a streamer at various distances from the limb, to determine the law of decrease with distance.

b. Measures of the relative heat radiation of rifts and streamers.

Of the four heat-measuring instruments available for this work the observer must endeavor to select that which is best adapted for his purpose. The radiometer and radiomicrometer are not affected by magnetic disturbances, and have other important advantages. The bolometer and thermopile require the

[1] "On a New Method of Mapping the Solar Corona without an Eclipse." This JOURNAL, 1, 318.

coöperation of an exceedingly sensitive galvanometer, but they are small and light, and can be used in any position. The thermopile needs no battery, and if made in Ruben's new form, as described in the last volume of the *Zeitschrift für Instrumentenkunde*, it ought to be well adapted for eclipse work. Each observer must, however, base his selection upon experience in the use of the various instruments. Whichever is chosen, means must be provided for instantly varying the sensitiveness through a large range, as with our present ignorance of its intensity it would be unsafe to expose a very delicate instrument to the full radiation of the corona. With a bolometer or thermopile a good balancing bridge is essential, and for the former it is necessary to have a battery giving a very uniform electromotive force. Every precaution must be taken to shield all junctions (which should be of copper throughout) from convection or radiation, and to protect all parts of the apparatus from the wind and other sources of disturbance. If possible, the measures should be differential, one strip, vane, or junction being exposed to the radiation from the Moon, the other to that of the corona. An observer with a single instrument should not attempt to measure the heat radiation at more than one or two points, using the lowest sensitiveness of the instrument that will give reliable readings. On account of the difficulty of the work, and the danger of a mishap at the critical moment, it is hoped that several observers with independent instruments may take part in it. The writer will be glad to discuss details of the apparatus (including heliostat, device for rotating coronal image, differential radiometer, radiomicrometer, bolometer or thermopile, galvanometer, balancing bridge, etc.) with anyone who is prepared to undertake the measurements.

It might be profitable to consider many other questions relating to the eclipse, particularly those connected with photometric and polariscopic observations. But, as stated at the outset, this article is intended to be merely suggestive in character, and no attempt has been made to cover the whole field. It will be

well worth while to undertake photometric observations of the corona, if photographic methods are employed in conjunction with some such instrument as Hartmann's new photometer. Work with the polariscope is less important at this time. Arrangements should always be made, if possible, to accurately determine the latitude and longitude of eclipse stations. The contacts should of course be recorded with all possible accuracy.[1]

It should perhaps be said in conclusion that the above suggestions do not necessarily represent the views of the Eclipse Committee, as the writer, except in a single instance, is alone responsible for them. The committee desires to secure for publication all possible information regarding the plans of eclipse parties, and communications may be addressed to the secretary at the Yerkes Observatory.

[1] Special attention is called to Professor Campbell's article on the observation of contacts in the *Astronomical Journal*, No. 474, which has appeared since the present paper was put in type.

RESEARCHES ON THE ARC-SPECTRA OF THE METALS.

V. SPECTRUM OF VANADIUM.[1]

By B. HASSELBERG.

VANADIUM AND THE OTHER METALS.

AFTER this comparison of vanadium with the metals I had previously investigated, I proceeded to a similar comparison of my observations with the spectroscopic investigations of other metals, made by Kayser and Runge. It appeared as a result of this comparison that several of these metals contained in their spectra no line in such position or of such character that any connection whatever with vanadium could be inferred. The metals are these :

Sodium,	Strontium,	Indium,
Potassium,	Magnesium,	Tin,
Lithium,	Zinc,	Lead,
Rubidium,	Aluminium,	Arsenic,
Caesium,	Gold,	Bismuth.

The metals

Calcium,	Mercury,	Thallium,
Barium,	Copper,	Antimony,
Cadmium,	Silver,	

show, however, occasional approximate coincidences with lines of vanadium, but in view of their diffuse character and the consequent considerable uncertainty of the wave-lengths given by Kayser and Runge, most of these are accidental and certainly not suitable as a basis for any inference as to a connection. These lines are given in the following tables along with the limits of error of the wave-length assigned by Kayser and Runge, with other remarks.

[1] Continued from Vol. X, p. 361.

λ	i	λ	i	Δλ	Remarks
V		Sb			
3637.95	1.2	3637.94	2.3	±0.03	
		Th			
3775.85	1.2	3775.87	6	±0.03	Reversed
		Ag			
4379.38	4.5	4379.45	2.3	±0.15	Broadened
		Cu			
3524.38	2+	3524.31	1.2	±0.10	Broadened
3533.85	3	3533.84	2.3	0.05	Broadened
4531.01	2+	4531.04	4.5	0.10	Reversed, broadened toward red
		Hg			
3543.68	1.2	3543.65	3.4	±0.10	Hazy
4916.48	1.2	4916.41	3.4	0.10	Hazy toward red
		Ba			
3910.01	3	3910.04	4.5	±0.05	Reversed
3975.51	1	3975.55	1.2	0.10	Hazy toward red
4179.57	2.3	4179.57	1.2	0.20	Hazy toward red
4332.98	3	4333.04	3	0.05	Hazy
		Cd			
3729.22	1.2	3729.21	3	±0.20	Broadened toward red
		Ca			
4240.53	2+	4240.58	3	±0.10	
4586.15	1.2	4586.12	6	0.10	
5260.56	1	5260.58	3	0.05	

Among these lines those of calcium in the last group are the only ones whose assignment to vanadium can be doubted. I have, nevertheless, retained them here because two of them have only a moderate intensity in calcium, and the third certainly corresponds to the solar line 4586.05, but not to the vanadium line. The observation made during the measurement that the last occupied a position somewhat to one side of the solar line also favors this view. The first two lines of the table indeed show a very accurate agreement with the corresponding lines of antimony and thallium; since nothing was seen, however, of the rest of the strongest lines of these metals in the vanadium

spectrum, we can hardly conclude that they occur as an impurity in the vanadium.

In addition to the investigations of metallic spectra by Kayser and Runge which have so far been utilized, a similar research on the spectra of the platinum group by Kayser[1] alone has recently appeared, which seems to give an accuracy in the positions of the lines not hitherto attained. The precision in fact appears to be of the same degree as that obtained by Rowland in the solar spectrum, so that it is justifiable to carry the third decimal of the tenth-meter. The accordance reaches this high degree not only in the measures among themselves, but also in comparison with those of Rowland; while in similar series of measures up to this time the individual values of an individual observer may present a similar accuracy, but the final results differ by several units of the second decimal place from those of other observers. It is evidently entirely illusory to carry the third place under such circumstances. That so sharp measurements could be obtained in this case must nevertheless be attributed not only to the superior dispersion of the large concave grating, but also, and perhaps chiefly, to the excellent character of the lines of the metals here investigated, which exceed in sharpness the lines of most of the other metals.

Since in my measurements of the vanadium lines the probable error of the wave-lengths must be set at about ± 0.02 tenth-meters, while in Kayser's determinations of the spectra of the platinum group the uncertainty affects only the third place, I have limited more closely the range of approximate coincidences in the comparison of my observations with his, and have only included those lines whose wave-lengths differ from each other at most by ± 0.05 tenth-meters; for it would seem quite certain the lines with a larger difference of wave-length are actually separated, and hence independent of each other. In this way I arrive at the following summary:

[1] *Abhandlungen der Berliner Akademie der Wiss., 1897. Anhang.*

V λ	i	Pt λ	i
3505.83	1	3505.85	1−
3687.61	2.3	3687.58	2+
		Pd	
4406.80	4.5	4406.76	3
5668.61	2.3	5668.60	1+
		Ru	
3493.34	1	3493.38	1+
3605.83	1.2	3605.79	1.2
71.37	2−	71.36	1+
72.54	2	72.53	1+
3676.86	3	3676.82	2−
3755.85	1+	3755.86	1+
3790.62	1.2	3790.65	3
3890.34	3	3890.35	2.3
3942.16	2	3942.21	2.3
3950.37	2−	3950.37	2.3
4118.73	2	4118.68	1+
21.13	1	21.15	1+
82.77	2.3	82.81	1−
4197.77	2−	4197.75	2.3
4226.78	2	4226.82	1−
4297.86	2.3	4297.89	4.5
4341.15	3	4341.20	1+
4426.17	3	4426.18	1−
4428.67	3	4428.62	2+
4531.01	2+	4531.03	2+
4814.28	2.3	4814.33	1−
4833.17	2	4833.16	1+
5657.11	1+	5657.13	1+
5725.90	2	5725.90	2.3

V λ	i	Rh λ	i
3775.85	1.2	3775.86	1.2
3806.93	2	3806.92	2.3
17.98	2	17 99	1−
18.37	3	18.35	3
28.66	3.4	28.62	1+
3888.50	2.	3888.47	1+
4506.77	2−	4506.82	1−
4922.60	1.2	4522.63	1+
5605.20	2.3	5605.21	1−
5626.27	2.3	5626.25	1+
		Os	
3930.19	2.3	3930.15	1−
3963.77	2	3963.77	2.3
4003.70	1.2	4003.65	1+
15.20	1+	15.20	1−
4097.09	1.2	4097.09	1−
4400.74	4	4400.75	1−
4529.80	2.3	4529.85	1−
4738.51	1.2	4738.51	1.2
		Ir	
3952.10	2	3952.10	1+
4048.77	1.2	4048.78	1−
4051.11	2.3	4051.07	1+
4257.54	2	4257.53	1+
4261.37	2	4261.41	1+

The number of approximate coincidences, particularly in the case of ruthenium, is greater than would have been expected in view of the narrow limits taken for the difference of wave-length. A direct comparison of the two spectra will alone decide whether or not these are actual coincidences and in certain cases are impurities in the vanadium spectrum due to this metal. Judging by the estimates of intensity in perhaps the majority of the cases the reverse is indeed equally probable, or that the vanadium occurs as an impurity in the platinum metals.

RESULTS OF OBSERVATIONS.

On the basis of what has preceded I have derived from my measures the following catalogue of the lines to be assigned to the spectrum of vanadium. The last column contains the corresponding determinations by Rowland, but those lines of his catalogue which do not occur in mine are given in the column of remarks with the designation R.

Vanadium λ	R	V	i ⊙	Remarks	Rowland	
3486.05		1	2			
89.64		1			89.65	1
——	3491.46					
93.34		1				
97.13		1			97.08	1
3498.23		1				
3501.65		1			01.61	1
04.57		1.2	1.2			
05.83		1+	1.2			
17.44		2+	2		17.44	1
——	3518.49					
20.18		2	2.3	Faint Ti Line		
24.38		2+	1.2	Also Fe		
24.89		1.2	?			
29.90		2—	3	At edge of ⊙ line	29.88	1
30.91		2	2			
33.85		3	1		33.82	1
——	3540.27					
43.68		1.2	—		43.63	1
45.34		2	2		45.33	1
45.52		1.2	—	R. 51.67 1	45.42	1
53.43		3	—		53.41	1
55.32		1.2	?			
56.42		1.2	—			
56.97		2.3	—	Very sharp; at edge of ⊙ line 57.03 (Fe)		
60.75		1+	1.2			
62.32		1	?			
63.59		1	?			
——	3564.68					
66.33		1.2	2	V? Also an inconspicuous Ti line		
69.11		1.2	2	V?		
71.18		1.2	—			

Vanadium λ	R	V	i	Remarks	Rowland	
3571.82		1.2	1.2			
73.69		1.2	—		73.65	1
74.92		1	1		74.92	1
75.26		1	1.2			
78.01		1+	2		78.01	1
81.00		1	—			
83.00		1+	—		82.95	1
——	3583.48					
83.84		1+	2		83.84	1
89.91		2+	2		89.89	1
92.15		2+	2		92.16	1
92.71		1+	—			
3593.48		2	2		93.52	1
3600.20		1+	—		00.17	1
05.75		1.2	—			
——	3609.02					
09.45		1.2	2	Very sharp; V? Also faint Ti line		
16.91		1+	—			
19.10		1	—	Very sharp		
22.82		1	1	V?		
——	3623.33					
36.09		2	—			
37.95		1+	1			
38.57		1	—			
39.21		1.2	1	R. 39.72 1 Pb	39.16	1
40.25		1+	—			
41.28		1.2	—			
44.05		1.2	—		44.04	1
44.88		1.2	1.2	Very sharp; ⊙ line fine Dpl. viol. Comp.		
45.77		1.2	—			
49.13		2—	1.2		49.06	1
——	3653.64					

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
3663.73		2.3	—		63.69	1
65.30		2	—		65.26	1
67.87		2.3	—	Diffuse	67.84	1
69.57		1.2	1	⊙ has a triple; the strong one in the middle = Fe		
71.37		2—	1	Coin. ? R. 71.84 1		
72.53		2	—	Diffuse; Ru 72.53	72.52	1
73.55		3	1	Diffuse		
75.85		2.3	1.2	Very sharp	75.84	2
76.86		3	—	Diffuse; Ru 76 82 2	76.81	1
———	3680.06			R. 80.05 1		
80.26		3	?	⊙ line ?	80.21	1
83.26		3	—	Fe 83.23 R. 83.60. 1 Pb	83.24	3
84.83		1.2	—			
86.40		2	2		86.39	3
87.61		2.3	3	Also a strong Fe line; other Fe lines are hardly visible in the region		
88.22		2.3	1.2		88.21	3
90.41		2.3	1		90.41	3
92.36		3	1		92.36	3
———	3695.19					
95.48		2.3	1		95.45	1
3696.00		3	1		95.99	4
3703.71		3.4	2	Also a faint Fe line R. 04.66 1		
04.85		3	1.2		04.83	7
05.19		2.3	1	R. 06.17 Ca, Mn	05.17	5
08.88		1.2	—	Very sharp	08.85	1
14.12		1	—			
15.62		2	2	Very sharp; Ni 15.61 1.2		

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
———	3720.09					
3722.15		1+	—		22.14	1
22.27		1	1+		22.33	1
22.76		2	?			
23.52		1.2	—			
27.49		2	2			
29.22		1.2	—			
32.88		2	1.2	Very sharp		
34.59		1.2	—			
38.15		1.2	—		38.13	1
38.93		2—	—		38.90	1
40.38		1.2	2		40.37	1
41.65		1.2	1.2		41.63	1
46.02		2—	—	Very sharp. At edge of the ⊙ line 46.06 (Fe)		
———	3747.10					
48.11		1+	—			
51.02		2—	1.2	Very sharp		
51.94		1+	1.2			
53.44		1+	—			
55.85		1+	—			
56.18		1	—	At edge of ⊙ line 56.21 (Fe)		
60.40		1	1	V?		
60.96		1	—			
63.30		2—	—			
64.96		1	—			
69.23		1+	?	Perhaps ⊙ line		
———	3770.13					
70.68		1	—			
71.11		2	1			
71.31		1	1			
71.87		1.2	—	Diffuse or dpl.		

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
3774.27		1+	—			
75.34		1.2	1	Weak. Coin.?		
75.85		1.2	—	Weak		
76.31		1.2	—	Weak		
77.31		1	—			
77.63		1	2	V?		
78.48		1	2	V?		
78.83		2.3	2	Very sharp	78.81	3
79.80	—	1.2	—			
81.54		1.2	—			
82.70		1	—			
84.84		1	—			
87.68		1	—			
90.46		2.3	—		90.45	2
90.62		1.2	2	{ Ru has 90.65 3; Cr has .61 2	90.59	1
93.76		2—	2			
——	3794.01					
3795.12		3.4	?	At edge of a strong ⊙ line		
3800.05		2.3	?		00.05	3
03.62		2.3	1		03.61	3
03.92		1.2	—			
04.05		1.2	—			
06.93		2	—	Beside the ⊙ line 06.86; R. 07.42 2		
07.64		2+	?	Beside the ⊙ line 07.68; R. 08.14 4	07.63	3
08.64		2.3	1	Very sharp		
13.63		3	?		13.61	4
15.65		2	—			
——	3815.98					
17.98		2—	1			

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
3818.12		1.2	—			
18.37		3	2		18.37	5
20.10		2+	—		20.09	4
20.41		1+	—	In the shade of a strong ⊙ line; R. 20.59 2		
21.63		2+	1		21.61	4
22.14		2.3	1.2			
23.00		2+	1.2		23.01	2
23.35		2+	1			
24.12		2—	—			
28.67		3.4	2		28.68	7
32.97		1+	—			
33.36		1+	1			
35.70		2—	—			
36.20		2—	—			
——	3836.65					
39.12		2	1	Coin.?		
39.53		2	—			
40.27		2—	?			
40.56		2.3	—	At edge of ⊙ line 40.60(Fe)		
40.88		3	1		40.87	6
42.03		2—	?	Coin.?		
44.58		2.3	1	Sharp	44.57	4
45.03		1	—			
47.46		2.3	?	Sharp. ⊙ has a group of fine lines; coin.?	47.45	3
49.48		2	—		49.43	2
51.32		1.2	—			
52.27		1	—			
55.50		3	2	⊙ line broad, dpl.? tripl.?	55.49	4
56.00		4	2		55.96	7
——	3856.52					
58.83		1.2	1.2	Coin.?		

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
3859.51		1.2	—			
62.37		2—	—	Sharp		
64.02		2—	—	Sharp		
65.02		3.4	2	At edge of ⊙ line	64.98	5
67.50		1	—			
67.77		2.3	2			
70.72		1+	—			
71.23		2	?	Falls in a broad band in ⊙		
73.80		1+	—			
75.22		3	2		75.20	5
—	3875.22					
76.05		2+	1.2			
76.21		2.3	2	Also an inconspicuous Fe line		
79.82		1.2	—			
84.04		1.2	—			
84.60		1.2	—			
85.00		1	1			
85.91		1+	1.2			
86.72		2	—	Sharp	86.69	2
88.23		1+	—			
88.50		2	—	Diffuse; weak		
90.33		3	1.2	Very sharp	90.30	4
91.27		2+	—	Diffuse; broad; weak R. 92.47 4		
93.03		3	—	Very sharp beside the ⊙ line 93.00		
94.19		2	2	Sharp; ⊙ line a close dpl.		
96.29		2	—		96.26	2
97.22		2	—			
—	3897.60					
98.15		3	3	Diffuse; weak. ⊙ line strong dpl. Fe	98.08	1

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
3899.30		1+	2	⊙ line dpl. { 99.30 Fe, .17 V		
3900.33		2.3	—	Diffuse		
01.30		2.3	—	Diffuse		
02.40		3.4	2		02.37	7
02.71		1+	—			
03.42		1.2	1			
04.63		1	—			
06.89		2	2	Sharp. Also a faint Fe line		
10.01		3	2.3	V dpl.; also ⊙ line	09.99	5
10.95		2	—	Sharp		
12.36		2.3	—	Very sharp		
13.03		2—	—			
16.55		1.2	2	V? R. 14.44 1; R. 19.60 1		
20.15		1	—			
20.65		1.2	—			
22.05		2—	—		22.02	1
22.58		2.3	1		22.55	3
—	3924.67					
24.84		2.3	—	Companion toward red	24.77	3
25.36		2+	2		25.35	3
30.19		2.3	—			
31.40		1+	—			
31.50		2	—	R. 33.77 3 Ca		
34.16		3.4	—			
35.28		2.3	—			
36.42		2	—			
37.68		2—	—			
38.35		2—	—			
39.48		2—	—			
40.75		1	—			
41.40		1.2	2	Coin.?		

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
3942.16		2	—			
43.77		2.3	—			
——	3950.10			R. 44.13 3 Al		
50.37		2—	—			
52.09		2	2	Sharp. Ir has 52.10 1+ R. 61.65 5 Al	52.07	1
63.77		2	2	V line somewhat displaced toward violet		
68.24		2—	—			
72.10		1+	—	R. 68.59 1 Ca		
73.49		1	1			
73.79		2—	?			
——	3977.89					
79.30		2—	—			
79.59		2—	—		79.54	1
80.66		2—	—			
84.75		2	2			
88.97		2	—			
90.71		3	1		90.69	5
92.95		3	2		92.92	3
97.30		1.2	1.2			
3998.87		3	—	Near 98. 80 (Ti)	98.85	3
4000.24		1	—			
03.10		1.2	1.2			
03.70		1.2	—			
——	4003.92					
05.86		2+	2		05.84	1
09.94		1	—			
11.45		1+	—			
15.20		1+	—	{ Several fine lines here R. 22.04 1		
23.50		2	2		23.51	1
25.46		1+	—			
30.04		1.2	1—	⊙ line?		
31.37		1.2	1			

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4031.98		2	—	Between { 32.12 Fe, 31.94 Mn	31.96	1
32.62		1.2	1.2	Coin. ?		
33.01		1+	—			
——	4034.64			R. { 34.62 2 Mn, 33.19 3 Mn		
35.77		2	2.3	{ 35.88 Mn, .77 V, .73 Co		
36.93		1	1			
40.46		1+	—			
41.72		2	—	Beside ⊙ line 41.80		
42.78		2+	?	Perhaps ⊙ line	42.76	1
47.05		1+	—			
48.77		2—	—			
51.11		2.3	?	Perhaps ⊙ line		
51.48		2.3	1		51.48	4
52.60		1	—			
57.21		3	—		57.21	2
60.97		1+	?	Coin. ? R. 57.96 1 Pb		
——	4062.60					
64.09		2.3	—		64.06	2
67.90		1.2	—			
71.67		2.3	2		71.66	2
72.30		2+	—			
——	4083.77			R. 77.85 1 Sr		
90.70		3	1.2		90.70	5
92.09		1.2	—			
92.54		2	2	Also in Mn	92.53	2
92.83		3	2			
93.65		2+	—			
94.42		2	—			
95.64		3	1		95.61	5
97.09		1.2	1			

Vanadium λ	R	i		Remarks	Rowland	
		V	⊙			
4098.54		2	1—	⊙ line ?	98.51	1
4099.93		3.4	2		99.92	7
4102.32		3	1	⊙ line ?	02.29	3
——	4103.10					
04.55		2	—		04.52	2
04.92		2	1			
05.32		3	1.2			
07.64		1.2	2.3	Close by the Fe line 07.65	07.60	1
08.36		2	?			
09.94		3.4	2.3	Close by the Fe line 09.95	09.91	7
11.92		4	2.3		11.92	5
12.47		1.2	1.2			
13.65		2.3	—		13.64	3
14.69		1.2	—			
15.32		3.4	2		15.31	7
16.64		3	1.2		16.63	9
16.85		1.2	1.2			
18.34		2.3	?	Coin.?	18.32	1
18.73		2	—	Beside 1871 (Fe)		
19.58		2	1		19.58	3
20.69		2	?		20.66	2
21.13		1+	?			
——	4121.97					
23.65		3	2			
24.23		2—	—		24.20	1
28.25		3.4	2.3		[1] 28.15	7
29.00		2	?			
31.32		1	—	Mn has 31.36; separated	31.30	1
32.13		3.4	2.3	⊙ line dpl. { 32.22 Fe / .13 V	32.12	6
33.92		2—	—			
34.61		3.4	2.3	⊙ line dpl. { 34.60 / .50	34.62	7
36.25		2	—			

Vanadium λ	R	i		Remarks	Rowland	
		V	⊙			
4136.52		2	—			
39.39		2	—			
41.96		1.2	—			
42.75		1.2	1.2			
43.02		1+	?			
49.02		1.2	—			
50.84		2—	—			
51.52		1	—			
52.81		2—	—			
53.49		1.2	?			
55.39		1	—			
56.00		1.2	?			
——	4157.95					
58.14		1+	—			
59.84		2.3	1		59.82	2
60.57		1+	—			
62.51		1	—			
69.40		1.2	1	Ti 69.46 near by		
71.45		2	—			
74.18		2	1		74.15	1
75.30		1	1.2			
76.83		1—	—			
77.02		1	—			
77.25		2—	1			
79.53		2.3	2			
80.99		1	2	V ?		
82.23		1.2	—			
82.74		2.3	—	R. 83.07 4	82.73	1
83.43		1+	—			
83.59		1+	1.2			
——	4185.06					
86.95		1	—			
87.82		1	—			

[1] Misprint; should be 28.25

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
4189.99		2.3	—	In a group of faint ⊙ lines Coin. ?	90.01	—
91.70		2.3	—			
94.17		1+	—			
97.45		1	1			
97.77		2—	1	Sharp		
4198.78		2—	2.3	Sharp		
4200.35		2—	—			
———	4202.19					
02.52		1+	1		02.51	2
04.67		1	?			
05.23		1+	1.2		05.20	2
09.98		2.3	1.2		10.00	5
10.55		1	—			
11.02		1	—			
16.52		1	?			
18.86		2	2	Very Sharp		
19.65		1.2	—			
21.17		1+	—	Diffuse		
22.49		1+	—			
24.30		2	—			
25.40		1	1		25.37	1
26.78		2	—	In shade of Ca line 26.90. R. 26.87 4 Ca		
———	4226.89					
27.90		2	1			
29.87		2	—			
32.62		3	1—		32.60	7
33.09		3	1.2		33.01	7
34.12		3	—		34.15	7
34.70		2.3	1		34.67	7
35.90		2.3	1		35.91	4
39.12		1.2	—			
40.25		2	1	Coin. ?		
40.53		2+	2	Fe 40.56. Separated		

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
4241.48		2	—			
47.46		1+	?	Coin. ?		
———	4250.96					
51.45		1	?	⊙ 51.49		
53.02		1.2	—			
55.60		1.2	—			
57.53		2	1	Very sharp ⊙ line, trace	57.52	4
59.46		2	1+	Very sharp	59.45	4
61.37		2—	2			
62.32		2	—		62.31	4
65.28		2	—	Broad, dpl. ?		
67.50		1.2	1.2			
68.78		3	1		68.79	0
69.92		2	1	⊙ line dpl. { 70.02. 69.90.		
70.49		2	—			
71.71		3	—		71.71	17
———	4271.92					
77.12		3	1		77.10	7
79.12		1.2	—			
83.06		2	—			
84.19		3	—		84.21	5
86.57		2	—	⊙ has 86.63 diffuse		
87.97		2	—			
91.46		2—	—			
91.97		3	—		91.98	1
———	4293.25					
96.28		2.3	—		96.27	7
97.86		2.3	—		97.84	7
98.17		2.3	2	V? Also inconspicuous Fe line. A dense group between 03.7 and 98.2; partly Ti		

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
4303.70		2—	?	R. 99.24 1	03.70	2
06.35		2.3	—			
07.33		2.3	—			
09.69		1.2	—			
09.95		3	?		09.95	7
12.56		1+	—			
14.06		1.2	—			
16.02		1	—			
——	4318.82			R. 18.80 2 Ca		
20.46		1+	—			
22.51		1+	—			
30.18		3	1			
32.56		1.2	—			
32.98		3	1		32.98	10
34.23		1.2	—			
36.29		1.2	—			
41.15		3	1		41.16	10
42.36		1.2	—			
43.00		2—	—			
——	4343.39					
53.02		3.4	1		53.04	18
55.09		2	?	⊙ line ?	55.14	4
56.10		2.3	1.2		56.10	4
57.60		1.2	—			
57.82		1	—			
60.75		1.2	—			
61.18		1+	—			
61.57		1.2				
63.48		1	1.2			
63.69		2	?		63.69	4
64.37		2	1+		64.38	4
65.92		1.2	—			

Vanadium λ	R	V i	⊙ i	Remarks	Rowland	
4367.24		1+	—			
68.25		2+	1			
68.76		1.2	1		68.76	4
69.25		1+	—			
——	4369.94					
73.40		2	1		73.38	6
73.99		2	1	⊙ dpl. { 73.95. 74.05.	73.98	3
75.47		2	1			
76.25		1	—			
78.06		2—	—	Diffuse		
79.38		4.5	2.3	Reversed	79.39	1
80.69		2—	—		80.72	4
84.07		1	—	R. 81.19 1		
84.37		1	—			
84.87		4.5	2.3		84.87	1
87.40		1.2	—			
90.13		4.5	2	Reversed	90.14	7
90.79		1	—			
——	4391.15					
91.84		1.2				
92.24		2	1		92.23	4
93.26		2—	?		93.26	3
94.01		2—	1		94.00	4
94.98		1.2	1			
4395.40		4.5	2	R. 97.39 1	95.38	10
4400.74		4	1.2		00.74	10
03.86		1.2	—		03.83	4
05.20		2.3	1	Very sharp R. 06.28 8		
06.80		4.5	2		06.80	8
07.85		4.5	3	⊙ dpl. { Comp. to V.=V Comp. to R.=Fe, Ti	07.80	8
——	4407.85					
08.36		4	2		08.37	5

Vanadium λ	R	T i	⊙	Remarks	Rowland	
4408.67		4.5	3	⊙ line dpl. { 08.58—Fe .67—V	08.66	5
12.30		2	—	Very sharp	12.30	4
16.63		3	1+		16.63	5
20.08		2.3	—			
21.73		3	1	⊙ line extremely faint	21.74	10
22.40		1.2	1			
23.32		1.2	—	}	23.37	8
23.41		1.2	—	}		
24.10		1.2	—		24.08	2
24.74		1.2	1		24.74	4
25.86		2	1	R. 25.59 1 Ca		
26.17		3	1			
28.68		3	1		28.68	5
29.95		3	—			
30.68		2	—			
34.80		2+	—			
——	4435.85					
36.31		3.4	1		36.31	7
38.02		3.4	1		38.00	7
41.88		3.4	1.2	Coin.? Also faint Ti line	41.85	2
43.52		2	—		43.51	4
44.40		3.4	1	Also faint Ti line	44.38	3
49.77		2.3	—		49.74	4
51.09		2+	1.2		51.07	4
52.19		4	1		52.18	8
52.91		2	?	⊙ line ?		
——	4456.05			R. { 54.94 Ca; 56.07 1 Ca		
56.68		2	—		56.67	3
57.65		3.4	3	⊙ line dpl. { 57.60 Ti, V .71 Mn	57.63	3
57.97		2.3	1	R. 58.91 1		
59.93		4	2		59.92	8

Vanadium λ	R	V i	⊙	Remarks	Rowland	
4460.46		4.5	2.3	⊙ line broad. Group ? R. 60.85 4	60.46	10
62.56		3.4	—	Beside ⊙ line 62.62 (Ni) R. 65.67 3	62.53	10
67.04		2	2	Coin. uncertain		
68.19		2.3	?		68.17	3
68.94		2	—		68.93	3
69.88		3.4	—	R. 70.87 1	69.87	7
74.21		3	—		74.21	7
74.89		3.4	—		74.90	7
76.06		2	—			
——	4476.19					
80.20		2.3	—	Very sharp	80.21	3
86.44		1	—			
89.06		3.4	2		89.10	7
90.95		2.3	2	Very sharp	90.98	4
91.35		1.2	—	Sharp	91.34	2
91.66		1	—		91.65	1
——	4494.73					
95.16		1.2	?	⊙ line ?		
96.26		3	—	Beside ⊙ line 96.32 (Ti)		
97.03		2+	3		96.23	5
4497.57		2	—			
4501.01		2—	—	Diffuse R. 01.41 1 (Ti)	97.57	5
02.12		3	—		01.00	2
06.30		2	—		02.12	4
06.41		1.2	—			
06.77		2—	1		06.74	
08.11		1	—			1
——	4508.46					
09.49		2	1	⊙ line extremely faint	09.46	2
11.64		2	—		11.61	2
13.79		2	—		13.79	2
14.36		2.3	2	Also Fe, Co	14.36	4

Vanadium λ	R	V	⊙	Remarks	Rowland	
4515.74		1.2	—		15.73	1
17.77		2	2	Fe 17.70	17.74	3
20.31		2—	—	Diffuse	20.33	2
20.67		1.2	—	Diffuse	20.68	2
24.38		3	—		24.38	5
25.31		2+	2.3	Sharp. Also Fe	25.34	2
28.16		2.3	—		28.17	3
28.66		2—	—			
29.47		2+	—		29.48	2
29.76		2.3	—			
30.97		2+	—	Beside ⊙ line 30.91 (Cr)	30.97	3
———	4534.95			R. 34.11 3		
37.84		2	1		37.83	4
40.18		2	—	Perhaps ⊙ line	40.18	4
41.57		1	—			
45.57		3.4	?	⊙ line ?	45.57	10
49.81		3	2.3		49.82	8
52.05		2	—		52.02	2
53.25		2.3	—	R. 52.73 5		
———	4554.21					
60.90		3	?	⊙ line ?	60.89	7
64.76		1+	?	⊙ line ?	64.76	1
70.60		2	—			
71.96		3	—		71.96	5
77.36		4	1		77.35	7
———	4578.73					
78.92		3	?	⊙ line	78.91	5
79.38		2.3	—		79.37	2
80.57		4	2		80.56	8
				R. 81.41 1		2
83.96		2	—	Beside ⊙ line 84.02 (Fe)	83.97	
86.15		1.2	—	Beside ⊙ line 86.05 (Ca)		
86.54		4.5	1		86.55	8
88.94		1+	—			

Vanadium λ	R	V	⊙	Remarks	Rowland	
4591.39		2.3	—		91.41	5
4594.27		4.5	2		94.22	10
4600.34		1.2	—			
———	4602.18					
06.33		2.3	—		06.32	4
07.40		1.2	—	R. 08.63 1	07.39	1
09.84		2	—		09.82	4
11.10		1.2	—		11.10	1
11.92		2—	—			
14.08		1	1.2	V ? R. 13.98 1	14.09	1
16.18		1	—		16.19	11
17.03		1	—			
18.00		1	—			
19.85		2	?	⊙ line ?	19.90	0
19.97		2.3	?	⊙ line ?		
21.43		1	—		21.43	1
24.62		2+	?		24.58	4
26.67		2+	1.2		26.67	4
———	4629.51					
30.24		1	—		30.24	1
35.35		2.3	?	⊙ line ?	35.35	7
36.34		1.2	—		36.34	1
40.25		2	—		40.24	5
40.92		2	—		40.91	6
44.64		2—	?	⊙ line ?	44.62	2
46.17		1+	—		46.16	1
46.59		2.3	?	⊙ line ?	46.57	8
48.08		1	1.2	Coin. uncertain	48.05	1
———	4648.83					
49.08		1.2	1		49.07	2
53.15		1	—		53.11	1
54.84		1.2	—	⊙ has { 54.80 Fe .91 Cr		
55.47		1	?	Perhaps a fine ⊙ line	55.41	1

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4657.17		1	1.2		57.14	1
61.01		1	—			
62.02		1	—			
63.33		2+	1.2	⊙ line dpl. Viol. comp. R. 62.61 1 Coin. with R. 63.31 3		
—	4668.30					
69.50		1+	1		69.49	1
70.66		4	—		70.67	8
72.48		1	2	V?		
73.83		1	—		73.84	1
79.65		1	—			
79.95		1.2	—		79.96	1
81.07		1.2	1	Coin. uncertain	81.07	1
82.09		1+	1.2	V?		
—	4683.74					
84.64		2	—		84.63	3
87.10		2.3	—		87.10	5
88.24		1	—			
90.45		1	—		90.44	1
4699.52		2—	2		99.51	2
—	4703.18			R. 02.69 1		
4705.26		2	—		05.28	3
06.34		2.3	—		06.36	5
06.75		2.3	1.2		06.76	5
07.62		2	2	R. 08.40 1	07.63	3
10.74		2.3	—	R. 09.13 1	10.75	5
13.61		1.2	—		13.64	1
14.28		2.3	1.2			
15.61		1.2	—	R. 15.49 1 (Ti)	15.65	1
16.08		2+	—		16.08	4
16.36		1.2	—		16.38	1
17.85		2.3	—		17.87	5
21.42		1.2	—		21.44	1

Vanadium λ	R	V (i)	⊙ (i)	Remarks	Rowland	
4721.70		2.3	—		21.70	4
23.06		2.3	—		23.06	4
23.65		1	—		23.63	1
—	4727.63			R. 24.07 1		
28.85		1+	—		28.84	1
29.73		2+	—		29.72	5
30.57		2	—		30.57	2
31.42		1.2	—		31.44	1
31.74		1.2	—		31.74	1
32.12		1.2	—		32.11	1
37.91		1+	—		37.92	1
38.51		1.2	—	Os has 38.51 1+	38.51	1
39.79		1	—		39.85	1
42.79		2	1		42.82	5
46.81		2	—		46.83	5
47.30		1.2	—		47.31	1
48.70		2+	—		48.72	5
51.16		2+	—		51.21	5
51.45		1	—		51.46	1
51.75		2	—		51.76	5
54.13		2.3	—	R. 52.04 1 ⊙ has 54.23 (Mn)		
—	4754.23					
57.55		2	—			
57.68		2.3	—	⊙. has 57.77 (Fe)	57.69	4
58.92		1.2	—	R. 59.21 1	58.94	1
65.84		1.2	—	R. 64.22 1	65.86	1
66.80		2.3	1	R. 69.21 1	66.84	7
72.74		1+	—		72.78	1
73.25		1.2	—		73.26	1
76.54		3	1.2			
76.70		2+	1.2		76.64	5
—	4783.60			R. 81.51 1		
84.65		2	—		84.66	5

Vanadium λ	R	V	⊙	Remarks	Rowland	
4786.70		3	3		86.71	7
93.10		2	—	R. 89.10 1	93.13	2
95.27		2—	—	R. 94.73 1	95.29	2
97.07		3	1		97.12	8
98.12		1.2	—		98.15	1
99.20		1+	—		99.21	1
4799.94		2+	1.2		99.97	4
———	4805.25			R. 03.24 1 / 02.37 1		
4807.70		3.4	1		07.74	10
19.22		1.2	—	R. 08.84 1	19.22	2
———	4824.32			R. 23.03 1		
27.62		3.4	1		27.64	10
29.00		1.2	—		29.01	1
30.86		1.2	—	R. 29.43 1	30.88	1
31.80		3.4	1+		31.84	8
32.59		3	1		32.62	8
33.17		2	—		33.21	3
				R. 34.00 1 / 34.26 1 / 35.04 1		
43.16		1.2	—		43.19	2
				R. 46.80 1		
48.98		1.2		R. 49.26 1 / 49.46 1	49.00	1
51.65		4	1.2		51.69	10
				52.15 1 / 54.11 1		
59.34		2—	1	⊙ has 59.32 R. 57.24 1		
———	4859.93			Coin. ? 58.81 2		
62.83		2	1	Coin. ?	62.80	4
64.93		4	?	Coin. ?	64.94	10

Vanadium λ	R	V	⊙	Remarks	Rowlan	
				R. 70.33 1		
4871.46		2—	—	⊙ has 71.51 (Fe) No coin. with V	71.45	3
75.66		4	1+	R. 73.17 1	75.67	10
80.77		2.3	—		80.75	6
81.75		4	1.2		81.75	10
85.86		2—	—	R. 82.36 2	85.83	2
87.02		2—	—		86.99	2
90.32		1.2	—		90.26	1
———	4890.94					
91.43		1.2	—		91.41	2
91.81		2—	—	⊙ has 91.68 (Fe)	91.77	3
4891.43		2—	—		94.40	3
4900.84		2.3	—		00.82	3
———	4903.49					
04.59		3	2	Dpl. Coin. exact ?	04.58	5
05.10		1.2	—		05.05	3
06.06		1	—	R. 07.05 1		
08.92		1	—	R. 13.28 1	08.88	1
16.48		1.2	—	R. 19.17 1	16.44	1
22.60		1.2	—		22.54	1
———	4924.95					
25.83		2.3	—	⊙ has 25.75 (Ni)	25.84	7
32.24		2	1		32.21	3
33.82		1+	—		33.79	1
———	4942.66					
4943.04*		1.2	—			
5002.54		2+	—		02.51	2
———	5014.42					
14.83*		—	—	R. 47.84 1	14.81	4
60.91		1	—	51.78 1	60.83	1
5064.32		1.2	—		64.30	1
———	5064.83					
5105.37		1.2	—		05.32	2

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
——	5109.82					
5128.71		2.3	—	R. 37.77 1	28.71	7
38.58		2+	—		38.60	4
39.74		2—	?	⊙ line ?	39.70	2
——	5141.92					
48.95		2	?	⊙ line ?	48.89	
57.27		1—	—	R. 59.44 2		
59.56		2	—		59.52	2
65.14		1	—		65.07	1
——	5165.59					
67.04		1	—	R. 69.13 1	66.96	1
70.15		1+	—		70.11	1
72.35		1—	—	R. 74.71 1 } 76.68 1 }	72.28	1
77.03		2—	—		76.96	1
78.75		1	—		78.73	1
79.35		1	—		79.28	1
81.01		1	—		80.93	1
83.07		1—	—		83.03	1
——	5183.79					
92.22		1	—		92.19	1
93.18		2	2	V?	93.18	5
93.82		1+	—		93.80	1
95.01		2+	—		95.02	6
5195.58		2	2		95.56	2
——	5202.48			R. 97.22 1 } 00.52 1 }		
5206.82		1+	—		06.79	1
07.89		1—	—		07.84	1
12.47		1-	—		12.40	1
13.87		1+	—		13.84	1
16.80		1.2	—		16.77	1
——	5217.56					
25.97		1.2	—		25.92	3
33.91		1	—		33.90	2

Vanadium λ	R	i V	i ⊙	Remarks	Rowland	
5234.31		2	—	Very sharp	34.25	7
40.40		1	—		40.36	2
41.06		2	—	Very sharp	41.06	3
——	5242.66			R. 58.31 1		
60.56		1	—		60.53	1
61.20		1	—		61.15	1
66.33		1	—			
——	5270.50					
71.28		1	—		71.12	1
72.92		1	—			
82.75		1	—			
5287.88		1	—	} Several fine lines		
——	5288.71					
5302.40		1	—	} Several fine lines here. V?		
——	5307.55					
29.05		1	—			
30.65		1	—		30.62	1
——	5333.09			R. 38.81 1 } 58.62 3 }		
83.68		2--	—	⊙ has 83.58 (Fe)	83.65	1
85.39		2	—			
88.56		1.2	1	Coin. with a fine ⊙ line	88.53	1
——	5397.35					
5398.13		1.2	—			
5402.17		2.3	—		02.15	5
——	5415.42					
15.51		2.3	?	Fe has 15.42	15.48	5
18.33		1.2	—		18.32	1
20.32		1—	—			
21.96		1	—	R. 24.28 2 (Fe)		
34.43		2	—		34.41	4
——	5434.74					
37.93		1.2	—		37.89	1
43.50		1+	—		43.47	1

Vanadium λ	R	V i	⊙	Remarks	Rowland	
——	5447.13					
5458.39		2—	—	R. 55.03 1 Ru has 55.02 3.4		
——	5463.17					
64.30		1—	—			
68.05		1	—		68.03	1
71.56		1+	—		71.56	1
——	5477.13					
87.48		1.2	—		87.45	1
88.18		2+	—		[1]88.31	4
5490.22		1.2	—		90.18	1
——	5497.73					
5505.13		1.2	—	06.10 1; 07.74 1; 08.86 1	05.10	1
07.97		2.3	—			
				15.30 1; 17.44 1; 34.06 1; R. 35.08 1; 35.66 1; 42.95 1; 45.10 1		
11.41		1.2	—		11.41	1
——	5513.21					
46.18		2—	—		46.16	1
47.31		2.3	—		47.31	4
48.41		1+	—		48.40	1
57.71		1+	—			
59.00		2—	—		58.99	1
61.92		2—	—	R. 66.16 1; 67.70 1; 76.75 1; 84.60 1	61.90	1
——	5569.85					
84.75		2.3	—		84.75	5
85.00		1.2	1+	V ?	84.98	1
86.26		2	—		86.23	1
88.71		1.2	—		88.71	1
92.67		3	—		92.67	4
5593.22		1.2	—	R. 94.73 1; 98.05 1	93.21	1
5601.63		2—	—		01.63	1

Vanadium λ	R	V i	⊙	Remarks	Rowland	
——	5603.10					
5604.44*		1.2	—		04.44	1
04.91*		1	—		04.88	1
05.20*		2.3	—		05.19	4
22.34		1.2	—		22.32	1
24.80		2.3	—	R. also has 24.45 1	24.85	7
25.16		2	—		25.12	4
26.27		2.3	—		26.27	7
27.86		3.4	1		27.89	7
32.73		1+	—		32.70	1
——	5634.17					
—		—	—		34.14	1
35.76		1.2	—		35.74	1
46.36		2.3	—		46.35	5
——	5655.71					
57.11		1+	—		57.12	1
57.67		2.3	—		57.69	5
68.61		2.3	—		68.61	5
71.10		3.4	?	Perhaps a fine ⊙ line	71.09	10
——	5675.65					
83.47		1.2	—		83.45	1
88.02		1+	—		88.00	1
——	5688.43					
5698.74		4	?		98.76	1
5703.83		3.4	1		03.83	10
07.26		3.4	1.2		07.24	10
09.25		1.2	—		09.20	1
——	5709.69					
16.49		1.2	—		16.46	1
25.90		2	—	Sharp	25.88	3
27.25		4	1.2		27.30	10
27.90		2.3	—		27.90	5
31.48		3.4	—			
33.34		1.2	—		33.34	1

[1] Typographical error for 88.21 ?

* The asterisk indicates that the particular wave-length is deduced from the following ⊙ line.

Vanadium λ	R	V i		Remarks	Rowland		Vanadium λ	R	V i	⊙	Remarks	Rowland	
5733.63		1+	—				5784.64		2	—		84.65	1
34.26		2	—		34.25	2	86.42		2	—		86.41	1
37.28		3	—		37.31	5	5788.85		1.2	—			
——	5742.07						——	5791.21					
43.67		2.3	—		43.67	5	5800.17		1.2	—			
47.98		1	—	Sharp			07.40		2—	—			
49.13		2	—	Sharp			——	5809.44					
50.90		1.2	—				17.33		1.2	—			
52.99		1.2	—		52.99	1	17.80		2—	—			
61.70		1.2	—		61.67	1—	30.97		2	—	Diffuse		
——	5763.21						——	5831.83					
72.66		2+	—	Sharp	72.66	2	39.34		1	—			
76.95		2	—	Diffuse	76.93	1.2	46.56		2	—	Diffuse		
82.85		1+	—		82.85	1	5850.60		1	—			
83.14		1	—				——	5853.90					
83.76		1+	—		83.76	1							

I am well aware that among these lines several can be found which are due to other metals hitherto not recognized. In this category of suspicious objects probably belong those lines which are inconspicuous in the vanadium spectrum, but correspond to strong absorption lines in the solar spectrum. This seems to be a reasonable inference from the fact that vanadium is represented in the general solar spectrum by only faint and inconspicuous lines corresponding to the most intense lines of the metal, while the metallic lines of medium and slight intensities are entirely absent in the solar spectrum. Provisionally, however, and as long as their origin cannot be established, the exclusion of these seems to me not quite justifiable, as slightly so as in cases where the lines arouse a suspicion of foreign origin on account of their extreme faintness.

The probable error of the above wave-length may be said to be about ± 0.02 tenth-meters, as already remarked. Rowland's values are doubtless more accurate, and in general are probably certain to two places of decimals. A comparison of our determinations nevertheless yields an entirely satisfactory result, the differences exceeding 0.03 tenth-meters only in a few cases. Among 431 lines compared the following differences occur the number of times given under N.

Δ	N	Δ	N
0.00 t.-m.	80	0.05 t.-m.	23
0.01	120	.06	7
.02	85	.07	8
.03	75	.08	4
.04	29		

The number of cases where the deviation exceeds 0.03 tenth-meters constitutes about 16 per cent. of the whole.

In Rowland's catalogue of the vanadium lines several occur which may be easily recognized as due to foreign impurities. In the following table I have collected these with the corresponding values of the wave-length according to Kayser and Runge, or myself, the estimates of intensity by Kayser and Runge, being transformed to my scale:

Rowland λ	i	Kayser and Runge λ	i	Hasselberg λ	i	Element
3639.72	1	39.71	6			Pb
83.60	1	83.60	6			Pb
3706.17	1			06.16	2.3	Mn
19.05	1			19.04	2.3	Mn
3933.77	3	33.83	6	33.77	—	Ca
44.13	6	44.16	6			Al
61.65	5	61.68	6			Al
68.59	1	68.63	6	68.62	—	Ca
4033.19	3			33.18	10	Mn
34.62	2			34.60	10	Mn
57.96	1	57.97	6			Pb
77.85	1	77.88	6			Sr
4226.87	4	26.91	6			Ca
4318.80	2	18.80	5			Ca
4425.59	1	25.61	6			Ca
54.94	1	54.97	6			Ca
56.07	1	56.08	5			Ca
4501.41	1			01.42	3	Ti
4715.49	1			15.46	2	Ti
5424.28	2	24.27	6			Fe
5535.66	1	35.69	6			Ba

We find here the principal lines of calcium and of lead, as well as of aluminium, and among those lines not occurring with me, probably a few other lines of the platinum metals.

I have already remarked that vanadium is only slightly represented in the general solar spectrum, faint absorption lines corresponding only to the strongest lines of the metal. In order to give a survey of this I have collected in the following table all the stronger vanadium lines (i = 3 or above) along with the estimated intensities of the coinciding solar lines.

Vanadium λ	i	⊙	Vanadium λ	i	⊙	Vanadium λ	i	⊙
3533.85	3	1	3818.37	3	2	3998.87	3	—
3553.43	3	—	28.66	3.4	2	4057.21	3	—
3673.55	3	1	40.88	3	1	90.70	3	1.2
76.86	3	—	55.50	3	2	92.83	3	2
80.26	3	?	56.00	4	2	95.64	3	1
83.27	3	—	65.02	3.4	2	4099.93	3.4	2
92.37	3	1	75.22	3	2	4102.32	3	1
3696.00	3	1	90.34	3	1.2	05.32	3	1.2
3703.71	3.4	2	3893.03	3	—	09.94	3.4	2.3
04.85	3	1.2	3902.40	3.4	2	11.93	4	2.3
3795.12	3.4	?	10.01	3	2.3	15.32	3.4	2
3813.63	3	?	34.16	3.4	—	16.65	3	1.2

Vanadium λ	i	⊙	Vanadium λ	i	⊙	Vanadium λ	i	⊙
4123.65	3	2	4416.63	3	1+	4578.92	3	?
28.25	3.4	2.3	21.73	3	1	80.57	4	2
32.14	3.4	2.3	26.17	3	1	86.54	4.5	1
4134.60	3.4	2.3	28.67	3	1	4594.27	4.5	2
4232.62	3	1	29.95	3	—	4770 66	4	—
33.09	3	1.2	36.31	3.4	?	76.54	3	1.2
34.12	3	—	38.02	3.4	1	4786.70	3	3
68.78	3	1	41.88	3.4	1.2	4807.70	3.4	1
71.70	3	—	44.40	3.4	1	27.62	3.4	1
77.13	3	1	52.19	4	1	31.80	3.4	1+
84.19	3	—	59.65	3.4	3	32.59	3	1
4291.96	3	—	59.93	4	2	52.65	4	1.2
4309.95	3	?	60.47	4.5	2.3	64.93	4	1.2
30.18	3	1	62.56	3.4	—	75.66	4	1+
32.98	3	1	69.88	3.4	—	4881.75	4	1.2
41.15	3	1	74.88	3.4	—	4904.59	3	2
53.02	3.4	1	89.06	3.4	2	5592.67	3	—
79.38	4.5	2.3	4496.26	3	—	5627.86	3.4	1
84.87	4.5	2.3	4502.12	3	—	71.10	3.4	?
90.13	4.4	2	24.42	3	—	5698.74	4	?
4395.40	4.5	2	45.60	3.4	?	5703.83	3.4	1
4400.74	4	1.2	49.82	3	2.3	07.26	3.4	1.2
06.80	4.5	2	60.90	3	?	27.25	4	1.2
07.85	4.5	3	71.96	3	—	31.48	3.4	—
08.35	4	2	77.36	4	1	5737.28	3	—
08.67	4.5	3						

It cannot be doubted from this table that vanadium enters into the general composition of the solar atmosphere, but apparently in only slight quantities. In this respect the conditions are quite different from those prevailing in Sun-spots, in the spectra of which, according to a communication by Young, many lines of vanadium invisible in the general solar spectrum attain a considerable intensity and breadth. The occurrence of vanadium in the Sun is not at all surprising in view of its very wide dissemination among the terrestrial minerals, and this adds interest to the fact that the metal similarly enters into the composition of a number of meteorites. I have frequently had opportunity to observe this in investigations of the arc spectra of these bodies on which I am at present at work. I shall soon report more fully upon this question.

ON THE DEVIATIONS FROM THE LAW OF RECIPROCITY FOR BROMIDE OF SILVER GELATINE.[1]

By K. Schwarzschild.

The so-called law of reciprocity states that sources of light of different intensity I produce an equal degree of blackening in their photographic images under different exposures t if the product $I \times t$ has the same value in the different cases. Laboratory experiments by Abney, Miethe, and Michalke have demonstrated that there are deviations from the law of reciprocity. These, however, reveal themselves most clearly in astrophotographic work. Scheiner, in 1891, proved that the increase in the number of fainter stars on prolonging the exposure fell far below what would be expected according to the law of reciprocity. In determinations of stellar brightness by the photographic method I have recently been able to confirm once more the existence of such deviations, and to follow them up in a quantitive way, and to express them in the following rule, which should replace the law of reciprocity: Sources of light of different intensity I cause the same degree of blackening under different exposures t if the products $I \times t^{0.86}$ are equal. In these experiments Schleussner's gelatine emulsion plates were employed. The exposures ranged from 3 to 5000 seconds, the intensity from one to a thousandfold, the blackening from the slightest degree up to almost complete opacity.

The results of a research by A. Schellen[2] are contradictory to the above result, as he found the law of reciprocity exactly confirmed for these same Schleussner plates, although this was after he had brought them to the maximum of a preliminary exposure. The contradiction cannot be explained, however, by the absence of a preliminary exposure in my experiments, as I

[1] *Photographische Correspondenz*, 1899.

[2] "Ueber die Giltigkeit des Bunsen-Roscoe'schen Gesetzes für Bromsilbergelatine." Münster, 1898.

obtained the same deviations for slight as for heavy degrees of blackening, and an appreciable effect of preliminary exposure on intense blackening is not to be thought of.

It was therefore desirable to repeat the laboratory experiments. This was made possible by the courtesy of M. Eder, who placed at my disposal the necessary appliances in the *k. k. graphische Lehr und Versuchsanstalt*, and assisted me with many suggestions. The procedure consisted simply in making plates with different (continuous) exposures at different distances from the normal benzine lamp which is used with the Scheiner sensitometer. All of the plates to be compared were cut from the same plate and were developed in the same bath at the same time.

It will suffice here to give a specimen of the results.[1] Equal degrees of blackening were produced in one series of experiments by the following combinations of intensity I and the exposure t:

1		2	
I	$= 81$	I	$= 1$
t	$= 4.8$ secs.	t	$= 785$ secs.
$I \times t$	$= 389$	$I \times t$	$= 785$
$I \times t^{0.86}$	$= 312$	$I \times t^{0.86}$	$= 309$

We see how striking the deviation is from the law of reciprocity. On diminishing the intensity to $\frac{1}{81}$, twice the amount of light is necessary to produce the same blackening. The products $I \times t^{0.86}$, on the contrary, come out equal within the limits of the errors of observation. For the intensity 1 the first trace of blackening appeared with an exposure of 10 seconds, A preliminary exposure of this amount did not alter the results of the experiment, as was expected, and the most dissimilar developers gave the same deviation.

Thus the laboratory experiments, confirm the formula found from the star plates. The bromide of silver gelatine investigated had also the property of employing so much the less of the incident energy for the photographic work, the slower the influx of energy. The diminution of the action of the light

[1] The detailed account of my experiments will soon appear in the publication of the von Kuffner Observatory.

under intermittent exposure, which has been noticed by Abney and more recently by R. Englisch, and which I also found for these highly sensitive plates, is connected with this. In a simple experiment with the aid of Scheiner's sensitometer[1] equal degrees of blackening were produced by
continuous exposures of

96, 72, 48, 24, 12 secs.:

and intermittent exposures of

99, 80, 54, 30, 16.5 secs.

Here the single durations of exposure were to the period of inter ruption, as

1 : 2.5; 1 : 3.1; 1 : 4.6; 1 : 8.2; 1 : 15.0.

The following illustration, which may be useful as a "mechanical analogy" for the further discovery and formulation of the laws applying here, is intended to exhibit the peculiar behavior of the plate: Suppose that a kite, instead of being held by the cord, carries a rather heavy rope, the end of which trails over the ground. If the wind is strong (corresponding to a strong intensity of light) the kite will rise high, only a short portion of the rope will remain on the ground, and the kite will fly rapidly before the wind without much friction for a long distance (corresponding to a strong degree of blackening). If, on the contrary, the wind is light, the kite will remain low and the slight energy of the wind will be for the most part lost in the friction of the long extent of the rope lying on the ground. If the wind is intermittent the kite has so much more time to settle down after every rise, and the motion will be accompanied on the average by the greater friction the longer the intervals between gusts of wind (corresponding to the behavior of the plate under intermittent illumination.)

VON KUFFNER'SCHEN STERNWARTE,
Wien-Ottakring.

[1] This consists of a rotating disk, from which portions have been cut out so as to allow the light to fall on the plate only during a definite fraction of a revolution.

ON THE EFFECT OF INTERMITTENT EXPOSURE ON BROMIDE OF SILVER GELATINE.

By K. Schwarzschild.[1]

An intermittent exposure can be varied in the four following ways, with the limitation that the source of light is constant during the experiment and the interruptions of the exposure are of uniform interval:

1. By the change of the ratio of the pause to the single exposure q.

2. By the change of the single exposure and the following pause taken together t.

3. By the change of the effective intensity of the light J.

4. By the change of the total duration from the first to the last exposure T.

These four quantities, which were thus selected for the convenience of explanation, yield all the others which might subsequently be of interest, as for instance:

The duration of the single exposure, $t_1 = \frac{t}{1+q}$;

The duration of the pause, $t_2 = \frac{tq}{1+q}$;

The sum of the exposures, $T_1 = \frac{T}{1+q}$;

The number of interruptions, $n = \frac{T}{t}$.

The effect of intermittent exposure has been investigated for the bromide of silver gelatine emulsions of C. Schleussner in these fourfold relations.

The series of experiments A was carried out with the aid of the Scheiner sensitometer of the *k.k. graphische Lehr- und Versuchsanstalt* which was most kindly placed at my disposal for this purpose by M. Eder. In this instrument, as is well known, a disk rotates between the source of light and the plate with sections

[1] *Photographische Correspondenz*, 1899.

cut out of different breadth which are numbered and so calculated that the ratio q of the covered portions of the circuit to the free part, and hence the ratio of the pause to the single exposure, is determined by the formula

$$q = 1.28^{N+5} - 1.$$

The normal benzine lamp belonging to the sensitometer was always placed at a distance of 1 meter (intensity of light $J=1$), and the period of revolution of the disk, which was the same as the duration t of the pause plus the single exposure, always amounted to 0.08 secs. In the first series of experiments J and t were therefore constant. But q on the contrary varied for the different sections cut out of the disk, and the total duration of exposure T was varied from 1 minute to 12 minutes, so that from 720 to 8400 interruptions took place. One experiment ran as follows: A strip of the plate received a series of continuous exposures. Another strip of the same plate was illuminated for a certain time T through the rotating sensitometer disk, and then had a numerical scale of degrees of blackening. It could be easily calculated from the above formula to which sensitometer number each continuous exposure would have to correspond, if it depended only on the sum total of the time of exposure, and the interruption made no difference. In fact equality was observed to exist between the continuous exposure and a sensitometer number different from the calculated one; in case of only approximate equality the deviation being estimated in tenths of the scale interval. For example, in one experiment I gave the first plate the following continuous exposures T_1 and illuminated the second plate for 4 minutes in the sensitometer. The calculated sensitometer number is given under N, the observed under N'.

T_1	N	N'	$N-N'$
28.8 s.	3.9	3.6	0.3
14.4	6.8	5.8	1.0
7.2	9.7	8.2	1.5
4.8	11.3	9.5	1.8
2.4	14.2	12.4	1.8

The result of the entire series of such experiments is collected in the following table. The one argument is q, the ratio of the pause to the exposure, the other is the total exposure T in the sensitometer. The difference $N-N'$ between the calculated and the observed sensitometer number is given. A positive number indicates a stronger action of the continuous exposure than the intermittent exposure. A negative number indicates a weaker action. It should be remarked, further, that a falling off of 1 sensitometer number corresponds to a loss of something like 25 per cent. of the luminous energy.

q $T=$	1 m	2 m	4 m	6 m	12 m	Mean
1.5	—0.3	—0.1		—0.1		—0.2
4	0.1	—0.1		—0.2		—0.1
6.5	0.5	0.1	0.3	0.1	—0.4	+0.1
14	0.9	0.6	1.0	0.5	0.6	0.7
24	0.9	1.0	1.6	1.2	1.3	1.2
60		1.2	1.8	1.7	2.0	1.7
130				1.5	2.5	2.0
260					2.3	2.3

Result: (1) The effect of the intermittent action depends for a given q not at all or very little on the duration T of the whole exposure in the sensitometer. Since the blackening increases with the duration of exposure for each separate q, for each sensitometer number, we see that the degree of the final blackening is of slight importance in the phenomenon. We are always concerned here, moreover, with the region of medium degrees of blackening only, since the very strong degrees of blackening are too uncertain for comparison, and the very weakest are too much affected by accidental preliminary exposure of the plate. We may, therefore, take the mean of the values of each horizontal line and find: (2) the effect increases with q. The intermittent exposure falls further behind the corresponding continuous exposure, the longer the pause is in proportion to the single exposure. The small negative values for a small q, which would indicate a stronger action of the intermittent than a continuous exposure, are to be assigned to errors in the experiment, and only show that if the pause is not much longer than the

duration of the single exposure as many as 8400 interruptions (for $T=12$ min.) exert no demonstrable effect.

Strictly this same series of experiments should now be performed for all other possible systems of values of the intensity J and durations t of single exposure plus pause. Two simplifications, however, may be permitted. We may generalize *a priori* that it would not depend on the final degree of blackening for the other values of J and t, and can therefore limit ourselves to the test of a single blackening, which of course should be chosen of medium degree. Secondly we need not test all the ratios q of pause to exposure since the general increase of effect with rise in q is recognized, but we may limit ourselves to two extreme cases, for which I have selected $q=1$ (pause equals duration of single exposure) and $q=23$.

Series B. $q=1$. With the assistance of gears different velocities of rotation could be communicated by clockwork (a chronograph of the von Kuffner Observatory) to a disk from which twelve sectors of 15° aperture at equal intervals were cut out. The lamp was set up at a definite distance, and acted on the plate for a certain period with the rotating disk intervening and then for half the time without the disk, so that the sum of the exposure times was equal in the two cases. The degrees of blackening obtained were both estimated on a scale obtained with the Scheiner sensitometer and the difference of the estimates was taken. The following table with double argument gives the differences which were obtained in a series of experiments with different intensity J (distance of lamp 40, 100, 200, 450 cm) and with different velocities of rotation of the disk. Instead of the time of revolution its twelfth part is directly given under t, which expresses the sum of the single exposure and pause. Positive numbers indicate as above a stronger degree of blackening by continuous exposure.

t	$J=$ 4	1	$\frac{1}{4}$	$\frac{1}{20}$
0.02^s	0.2	—0.1	0.5	
0.4	0.0			0.3
1.8	0.3	0.1	0.4	
3.2			0.3	0.0
6.4				0.1

Result. For very different intensities and durations of a single exposure $\left(t_1 = \frac{t}{2}\right)$ only a very slight effect of the intermittence is indicated, on the average a diminution of 0.2 sensitometer numbers. Therefore if the pause is of the same length as the single exposure no appreciable effect is lost under the conditions here obtained. It is to be noted that the last three experiments ($t=3.2$ *s* and 6.4; therefore the single exposures of 1.6 and 3.2 secs.) were performed without the rotating disk by illumination with the hand according to a clock which beat fifths of a second. These prove therefore only that exposures from one to two seconds can be exactly made by hand.

Series C. $q=23$. This series ran like the others except that a disk was used which had only one sector cut out of $15°$ and the continuous exposure therefore had to be $\frac{1}{4}$ of the exposure with the rotating disk to produce an equal total duration of exposure. The following table gives the result of the experiments in this same way as before. The argument t, the sum of the pause and the exposure, correspond here directly to the time of rotation of the disk. The last experiments for $t=38$ secs. were carried out by illumination with the hand, but may be regarded from what has been found before as sufficiently certain.

Result. With a pause twenty-three times longer than the single exposure a heavy loss of luminous energy up to 40 per cent. (corresponding to two numbers of the sensitometer) occurs. The loss is the greater, the quicker the velocity of rotation, the shorter the single exposure, and an equal loss occurs with a slower rotation and longer single exposure if the intensity falls. Following up the behavior of this last more closely, we find at intensity 4 a loss of 1 sensitometer number for a rotation in $t=0.8$ secs., corresponding to a single exposure of 0.033 secs. With intensity of $\frac{1}{20}$ we find the same loss for a rotation of about 50 secs. corresponding to a single exposure of 2.1 secs. The quantity of luminous energy introduced during the single exposure is measured in the first case by $4 \times 0.033 = 0.13$, and in the second case by the closely similar value $\frac{2.1}{20} = 0.10$.

Therefore with varying intensity the same loss will occur when the amount of light introduced during the single exposure is the same. If this quantity reaches the so-called limiting value, the value which of itself produces a barely appreciable blackening, there will remain only a slight weakening of about 0.5 sensitometer units (12 per cent.) in consequence of the intermittence. For the 4 intensities this limiting value is respectively 0.1, 0.5, 2.5, 15 secs. Multiplying by 24 we obtain 2.5, 12, 60, and 360 secs., and if we enter the table with these values of t for the 4 intensities, we shall find on the average the above loss of 0.5 sensitometer numbers. We further see that the effect of the intermittence almost wholly disappears if the single exposure exceeds the limiting value by a multiple, and that it does not attain large values until the single exposure falls far below the limiting value.

With this all the variations of the intermittence have been experimented upon, and we reach the following conclusion. The effect of the intermittence depends upon two instead of four quantities; first, upon the ratio of the pause to the duration of the single exposure — the longer the proportion of pause the greater the weakening; and secondly, on the quantity of light which the single exposure sends to the plate. A definite ratio of the pause to the single exposure produces more weakening the further the quantity of light lies below the limiting value. On the other hand no appreciable influence is due to the degree of the blackening and the absolute magnitude of the exposure time, or the luminous intensity taken by itself.

The derivation of a quantitive formula from the above figures cannot be ventured, since the determination of the blackening by estimation on a scale must be considered a rather rough procedure in comparison with the delicacy of these effects. The qualitive character of the results, however, seems to be substantial, and is further confirmed by the experiments made several years ago by Abney.[1] He found that the retardation of the blackening under intermittent exposure was the more

[1] *Journal of the Phot. Society*, 1893–4, p. 63.

pronounced the longer the pause in comparison to the single exposure, the quicker the rotation of the disks employed, and the less the light intensity. Therefore if the intensity becomes stronger and hence the deviation less, we can evidently increase the deviation up to its original amount by raising the velocity of rotation, so that from Abney's experiments also the result, aside from the ratio of the pause to the duration of exposure, must chiefly depend upon the quantity of light admitted in the single exposure. Abney's experiments differ from those given above in that slow plates and shorter single exposures were employed. The more recent experiments of R. Englisch[1] would seem to indicate an effect of the same character.

The following experiment, which is included in the last table, is the most remarkable, and at the same time easiest to repeat. Let a fairly constant light be moved so far from the plate that in 15 secs. the first trace of a fog is produced (in the above case 450 cm) and with the eye on the watch expose with the hand and interrupt alternately 3 secs. and repeat this twenty times. At another place illuminate twenty times for 3 secs. with pauses of 1 minute. The second exposure will develop a markedly less degree of blackening than the first (in the above case 1 unit of the sensitometer) in spite of the equality of the total time of exposure. From this we may conclude that the process of weakening develops in a peculiarly slow manner in that it must complete its action wholly, or for the greater part, in the period from 3 secs. to 1 minute after the cessation of the exposure. It is particularly in view of this fact that it appears a quite difficult problem to explain the action of the intermittent illumination by other than an artificial hypothesis.

Remarks on the Scheiner sensitometer. It has been shown above that the action of a rotating disk itself changes with the velocity of rotation. In Scheiner's sensitometer the rotation is produced by hand, and is therefore not entirely uniform. Nevertheless the variation of the action with the velocity of

[1] Review in *Archiv für wissenschaftliche Photographie*, 1899, Heft 1.

rotation is so slow that the alterations which could be obtained by hand rotation anyhow must remain inappreciable, according to the above figures, and according to experiments of my own this is the case.

The difference in the action of continuous and intermittent exposure has, however, the further consequence that the original intention of this sensitometer, of constructing an accurate scale of exposure times at constant light intensity, cannot be obtained with the rotating disk. The higher numbers are too much weakened by relatively longer pauses. If, however, we should construct a scale of intensities by weakening the light in definite ratio, as, for instance, in the Warnerke sensitometer, instead of a scale of exposure time, this would deviate in an equal sense from the precise scale of the times of exposure, as the scale of Scheiner's sensitometer ; for, in consequence of the deviation of the reciprocity law above proven, a diminution of the intensity will produce a less blackening in the same proportion as a shortening of the exposure, and the higher numbers of the scale of intensity will again come out less than those of the exact time scale.

The most suitable thing in practice is properly something intermediate between a scale of time and a scale of intensity, because we have to deal neither with constant intensities nor constant exposures, as, for instance, highly sensitive plates are used now for instantaneous pictures of bright objects, and again for long exposures of faint objects. Scheiner's sensitometer therefore furnishes in general suitable data as to sensitiveness in practice just on account of its tendency from the time scale to the intensity scale which the intermittent exposure introduces. If the theoretical meaning of these data does remain perhaps indefinite and certainly complicated, it should be remembered that an exact determination of sensitiveness cannot be expressed by a single number in consequence of the deviation from the law of reciprocity and the different grades of plates.

Minor Contributions and Notes

CHANGE IN TIME OF PUBLICATION.

The attention of subscribers to the Astrophysical Journal is called to the fact that hereafter the February and August numbers of each year will be omitted, instead of the July and September numbers, as formerly. Thus the first volume ot each year will consist of the January, March, April, May, and June numbers, while the July, September, October, November, and December numbers will constitute the second volume.

THE GREAT REFRACTOR OF THE POTSDAM ASTROPHYSICAL OBSERVATORY.[1]

The recent dedication of the great refractor of the Potsdam Observatory was an event of the first importance in the progress of astrophysics. The principal address on this occasion was that of Director Vogel, who reviewed the advances which have been made in the determination of stellar motions in the line of sight, and referred to the important contributions to this work which we owe to the Potsdam Observatory. After Professor Vogel's address, the motions of the telescope and dome were explained and demonstrated by Professor Scheiner. The telescope has two objectives, one of 80 cm aperture and 12 m focal length, and another of 50 cm aperture and 12½ m focal length. Both objectives, for which the glass was furnished by Schott & Co., of Jena, were made by C. A. Steinheil Sons, of Munich; the larger of the two is corrected for the actinic rays, the smaller for the visual rays. The mounting by A. Repsold & Sons, of Hamburg, is of the so-called German form as modified by Repsold; the motions of the telescope in both coördinates are easily effected from the floor by means of two hand wheels supported on the column of the instrument. The weight of the moving parts is about 7000 kg.

[1] See frontispiece.

The dome is 22 m in diameter and 18 m high. The iron construction of the hemispherical movable part is by Bretschneider and Krügner, of Pankow; the inner lining of wood was put in place by Joester, of Potsdam. It rests on a system of twenty trucks, each containing three wheels, of which the middle one bears the dome, while the outer ones run on a track fastened to the masonry. The rotation of the dome, in which a weight of 200,000 kg is set in motion, can be effected by hand, without great labor, although very slowly; by the aid of electricity a complete revolution can be accomplished in five minutes. The driving mechanism was made by the firm of C. Hoppe, which also furnished the very ingeniously constructed movable platform for the observer. This movable platform, which was first suggested by Dr. J. Repsold, is suspended from the dome, with which it moves, directly opposite the observing slit. It can also be moved independently through a limited distance to the right and left. The stage on which the observer stands moves up or down on an inclined plane. This motion can be effected by hand, or with great ease from the platform itself by means of electric motors. The opening in the dome has a width of 3½ m and extends 1½ m beyond the zenith. The shutter can be operated by hand from a gallery on the inner wall of the tower or electrically from the observing platform. The lower part of the opening can be closed by means of two screens 5 m high, which are moved outward from the middle of the slit.

We are informed that the preliminary tests of the two spectrographs constructed for the new telescope by Toepfer have been in every respect satisfactory. It may be expected that the great work of determining the motions in the line of sight of some five hundred stars, for which the telescope is specially designed, will soon be in progress.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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